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An acquisition policy for a multi-supplier system with a finite-time horizon

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Abstract

We study the problem of a manufacturer who outsources a single product to multiple capacitated suppliers. Given a pool of potential suppliers, a decision has to be made about which supplier(s) to use, and how much and how often to purchase from each selected supplier. We build an EOQ-type model with a finite-time horizon, which differs from the existing model in the literature with an infinite-time horizon. We design a dynamic programming and a scatter search algorithm to solve the problem, and provide computational results to show the effectiveness of the proposed methods.

Keywords: Multiple suppliers; Capacity; Finite-time horizon; Scatter search; Dynamic programming

1. Introduction

Outsourcing is becoming an important business practice in many manufacturing areas. Supplier management is one of the fundamental issues for a successful implementation of outsourcing. While it is often suggested that a firm use a single supplier in order to build a long-term supplier relationship and improve the service quality, using multiple suppliers is still very popular in practice [1,2].

We refer to Minner [1] for a general review of research on multi-supplier inventory management. There have been some recent results in the literature. Basnet and Leung [3] considered a multi-product and multi-supplier lot sizing model, and developed an enumerative search algorithm and a heuristic method. Berger et al. [4] used a decision tree approach to address the issue of selecting an appropriate number of suppliers in the presence of risks. Crama et al. [5] discussed the problem of purchasing from multiple plants who offer total quantity discounts to produce multiple products with adjustable recipes. The decision involves both the strategic level (recipe) and operational level (supplier selection).

While a large body of papers have been published on handling multiple suppliers, we are particularly interested in two issues that have not been well addressed: the capacity constraint of the suppliers, and the finiteness of the planning horizon of the firm.

One of the reasons for using multiple suppliers is the capacity constraint. In outsourcing, especially for a labor-intensive product, the production is usually conducted in developing countries where there exist many suppliers with

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limited capacities that can provide similar services. Therefore, a firm, when relying on outsourcing for its supply, may have to deal with multiple suppliers, each with a limited supplying capacity.

Only a limited number of papers dealing with multi-supplier inventory management incorporated the issue of capacity into consideration. Kim et al. [6] built a single-period multi-item newsboy model with multiple capacitated suppliers. They also developed an iterative solution approach to solve problems with real-world data. Rosenblatt et al. [7] built a model (referred to as the RHH model) and developed an acquisition policy for a single-product multi-supplier system. They assumed a constant demand rate over an infinite-time horizon. The purchasing quantity from each supplier is limited by its periodic capacity. Their purpose was to determine the ordering frequency and quantity from each supplier so as to minimize the long-run average cost. This is essentially a variant of the classical EOQ model with an infinite-time period. They also proposed an approximation scheme so that a repetitive ordering policy could be implemented for a finite-time period. Literature related to the RHH model such as Haberl [8], Herer et al. [9], and Alidaee and Kochenberger [10] focused on developing efficient algorithms to solve the problem with an infinite-time horizon.

Another important issue is the finiteness of the planning horizon. In a classical EOQ-type model, the demand is assumed to last for an infinite-time horizon, which is an approximation of the real business world cases. Consider today's fashion market, for example, where the production life of a new design is typically less than one season. It becomes increasingly important to consider the effect of a finite planning horizon when making outsourcing decisions.

Under an infinite-time horizon, an order interval can be of any positive length. If the planning horizon is finite, however, the order interval is restricted by the planning horizon. Since no holding inventory is economical at the end of the horizon, the optimal order interval calculated in the infinite case might not apply to the finite one. For example, suppose that an optimal order interval under an infinite horizon is 12 days. If the planning horizon is 30 days, then the third order will result in inventory leftover after 30 days. So we have to adjust the order quantity, either for the third order or for all previous orders. This issue has been addressed in some early literature. Lev and Soyster [11] developed an inventory model with a finite demanding horizon and price changes. They also provided the form of the optimal policy and sensitivity analysis in terms of the length of the time horizon. Shwartz [12] studied the EOQ problem under the assumption that the demand horizon is finite, and developed a policy which resulted in significant cost savings compared to alternative policies. Both papers showed that, under the settings of a single supplier, it is optimal to let each order interval be of the same length. In other words, the whole planning horizon should be divided into integer number of equal periods. This conclusion can be readily extended to multi-supplier scenarios, and provides the basis of our modeling.

In this paper, we study an outsourcing problem where multiple capacitated suppliers are desired to satisfy the demand in a given period. Our research is directly motivated by the RHH model. We study the same problem except that the planning horizon is a given finite period. We solve the finite-time-horizon model directly rather than design an approximation based on an infinite-time-horizon solution.

The rest of the paper is organized as follows. In Section 2, we introduce our assumptions, notation and mathematical formulation. In Section 3, we review and make comments on an approximation approach based on the RHH model. Then in Sections 4 and 5, we discuss the design of dynamic programming and scatter search algorithms to solve our problem. Finally, the computational results, including cost savings and performance of the algorithms, as well as future directions of research, are presented.

2. Model description

We use an EOQ-type model with a finite planning horizon to formulate our problem. Most assumptions are similar to the RHH model except that the planning horizon is finite in our model. Without loss of generality, we assume that the planning horizon is one year. Suppose that we need to purchase a single product from multiple candidate suppliers, each of which has a fixed annual supplying capacity. The demand of the product occurs continuously within the planning horizon at a constant rate, and has to be fully satisfied without backlogging. Orders are placed in batches from the suppliers, and the cost of each order is setup-plus-linear. In addition, each supplier that is used (obtaining at least one positive order) causes an administration cost regardless of ordering frequency and quantity. Inventory cost is a linear function. The goal is to determine from which supplier(s) we should purchase the product, and the ordering frequency and quantity, so that the total cost is minimized.

We use the following notation to describe our problem:

M	number of suppliers
i	index for candidate suppliers, $i = 1, 2, \ldots, M$
D	the annual demand rate for the product
c_i	the unit purchasing cost from supplier i
s_i	the setup cost when an order is placed to supplier i
h_i	the annual cost for holding one unit product from supplier i
b_i	the annual supplying capacity of supplier i
F_i	the administration cost incurred if supplier i is to be used

We also use the following variables to define a solution to our problem:

d_i	the annual ordering quantity from supplier i
$\mathbf{d}$	purchasing profile for all suppliers, $\mathbf{d} = (d_1, d_2, \ldots, d_M)$
Q_i	the quantity of a single order from supplier i
n_i	the number of orders placed to supplier i
y_i	a binary indicator of whether supplier i is used. $y_i = 1$, if $d_i > 0$, and 0, otherwise.

When we order $d_i > 0$ units from supplier i by n_i orders, the resultant cost can be obtained from an EOQ model such as

$$f_i(d_i, n_i) = c_i d_i + s_i n_i + \frac{h_i d_i^2}{2Dn_i}, \tag{1}$$

where the first term is the variable ordering cost, the second term is the order setup cost, and the last term is the inventory cost. If $n_i = d_i = 0$, then we define $f_i(0, 0) = 0$.

Our objective is to minimize the overall cost over the planning horizon, i.e.,

$$\min_{d_i, n_i} \quad \sum_{i=1}^{M} f_i(d_i, n_i) + F_i y_i \tag{2}$$

$$\text{s.t.} \quad \sum_{i=1}^{M} d_i = D, \tag{3}$$

$$0 \leqslant d_i \leqslant b_i y_i, \quad i = 1, \ldots, M, \tag{4}$$

$$0 \leqslant n_i \leqslant N y_i, \quad i = 1, \ldots, M, \tag{5}$$

$$y_i \in \{0, 1\}, \quad n_i \in \{0, 1, 2, \ldots\}, \quad i = 1, \ldots, M. \tag{6}$$

In constraint (5), N is a very large positive number, which forces $n_i = 0$ when $y_i = 0$.

In the above model, there are two decision variables for each supplier, d_i and n_i, which makes the problem hard to analyze. In fact, we can see that once d_i is given, the optimal n_i^* can be uniquely determined from a finite-horizon EOQ model (e.g., [12]). Specifically, n_i^* can be calculated from the following formula:

$$n_i^* = \min \left\{ m : m(m+1) \geqslant \frac{h_i d_i^2}{2s_i D}, \ m = 0, 1, 2, \ldots \right\} \tag{7}$$

We only need to find the optimal d_i^* for each supplier i. Once d_i^* is given, n_i^* is uniquely determined.

3. Preliminary: an approximate scheme based on an infinite-horizon model

In this section, we briefly review an approximation approach based on the RHH model, make some comments on the existing results, and discuss potential improvements.

Table 1
Parameters of suppliers for Example 1

Supplier No. i	c_i	s_i	F_i	h_i	b_i
1	1.13	69.46	109.85	1.0	940
2	1.01	187.06	72.33	1.0	1060
3	1.44	153.30	194.94	1.0	500
4	2.57	73.84	145.09	1.0	870
5	2.86	107.46	163.56	1.0	750
6	1.61	201.25	118.56	1.0	740
7	1.91	170.73	150.16	1.0	410

In the RHH model, the planning horizon is assumed to be infinite. An optimal solution can be obtained in terms of d_i, and Q_i for each i. However, for a finite planning horizon τ, $\tau d_i/Q_i$, the number of orders for supplier i, may not be an integer, which makes it difficult to implement. In order to obtain a feasible implementation, they proposed an approximation scheme of rounding such that a new order quantity Q'_i can be found and $\tau d_i/Q'_i$ is guaranteed to be an integer for a planning horizon τ. They proved that the resultant new plan, referred to as the RHH solution, has a cost that is less than $(\sqrt{2}+1/\sqrt{2})/2 \cong 106\%$ of the cost of the optimal infinite-horizon solution provided that τ is large enough. This also implies that the RHH solution has an error bound no more than 106% from the optimal finite-horizon solution for a large τ. Our model, as described by (2)–(6), differs from their model in that we specify that the number of orders during the whole planning horizon has to be an integer. This new constraint makes our model directly applicable to the finite-time-horizon cases.

We now use several numerical examples to demonstrate the difference between the RHH solution and the optimal solution we are to find. We use TC_Q to denote the annual cost of the RHH infinite-horizon solution, $TC_{Q'}$ the annual cost of the RHH approximation solution, and TC_n the annual cost of our optimal solution.

Example 1. Consider the problem defined by the parameters in Table 1, with annual demand rate $D = 1000$.

The RHH solution is $\mathbf{d} = \{940, 60, 0, 0, 0, 0, 0\}$ with $TC_{Q'} = 1845.91$, and the corresponding infinite-horizon solution is $TC_Q = 1688.45$. In our finite-horizon model, the optimal solution is $\mathbf{d}^* = \{0, 1000, 0, 0, 0, 0, 0\}$ with $TC_n = 1704.65$. Compared to the optimal solution, the total cost obtained with the RHH solution is 8.27% more than the optimum.

This example also shows that the error of the RHH solution may be greater than 6% if the planning horizon ($\tau = 1$ in this example) is not large enough, indicating that the RHH solution may not be effective for any given planning horizon. Before presenting our method for approaching the problem, we use two more examples to show some insights for the RHH solution.

As an approximation scheme, the error of the RHH solution first comes from the rounding scheme, i.e., deriving Q' from Q^*. In addition, there are two other sources that may cause the RHH solution to differ from the optimal finite-horizon solution, an incorrect annual ordering quantity from each supplier, and an incorrect set of suppliers to be selected. In an RHH solution, the annual ordering quantity for each supplier is determined by making use of the following property of an optimal solution for an infinite-horizon problem.

Property 1 (*Rosenblatt et al. [7]*). *There exists an optimal solution to an infinite-horizon problem where there is at most one supplier who gets an order that is different from his capacity. Any other suppliers either get a zero order or a full-capacity order.*

We now show that such a property does not generally hold for a finite-horizon problem. Therefore, the RHH solution may have an incorrect order quantity for each supplier, as indicated by the following example.

Example 2. For the problem defined in Table 2 with annual demand $D = 1000$, the RHH solution is $\mathbf{d} = \{0, 20, 0, 0, 0, 0, 980\}$, with an annual cost $TC_{Q'} = 1876.93$. The optimal finite-horizon solution is $\mathbf{d}^* = \{0, 447, 0, 0, 0, 0, 553\}$ with $TC_n = 1773.46$. We can see that in the optimal solution, neither the order quantity from supplier 2 nor supplier 7 reaches its capacity. This is not consistent with Property 1.

Table 2
Parameters of suppliers for Example 2

Supplier No. i	c_i	s_i	F_i	h_i	b_i
1	1.83	83.86	182.32	1.0	580
2	1.17	154.80	59.11	1.0	850
3	2.00	123.01	182.62	1.0	730
4	1.41	132.17	153.29	1.0	490
5	1.98	89.20	178.47	1.0	950
6	1.12	137.11	129.81	1.0	970
7	1.06	160.82	36.38	1.0	980

Table 3
Parameters of suppliers for Example 3

Supplier No. i	c_i	s_i	F_i	h_i	b_i
1	1.66	127.84	196.38	1.0	500
2	2.14	125.66	90.96	1.0	450
3	1.85	168.71	27.32	1.0	600
4	1.73	204.54	69.53	1.0	1000
5	1.73	105.46	90.78	1.0	340
6	2.54	92.01	80.97	1.0	380
7	1.53	195.56	119.27	1.0	920

Moreover, the RHH solution may not only obtain the incorrect order quantity from each supplier, but also choose an incorrect subset of suppliers that are different from our optimal solution. This is shown in Example 3.

Example 3. The problem is defined in Table 3 with annual demand $D = 1000$. For the RHH solution, we have $\mathbf{d} = \{0, 0, 80, 0, 0, 0, 920\}$ with $TC_{Q'} = 2474.14$, while the optimal solution is $\mathbf{d}^* = \{0, 0, 0, 0, 340, 0, 660\}$ with $TC_n = 2381.06$. Besides the cost difference, we see that suppliers 5 and 7 are selected in the optimal solution, but suppliers 2 and 7 are selected in the RHH solution.

While the rounding error of the RHH solution may be controllable by more sophisticated schemes, the other sources of error, the set of selected suppliers, and the order quantity for each selected supplier, are fundamental. It follows that, in some cases, the RHH solution can never achieve the actual optimal solution, however the rounding procedure is improved. Therefore, new models and algorithms are needed for seeking the optimal finite-horizon solution in a more effective way.

4. A dynamic programming algorithm

4.1. A basic formulation

We use a dynamic programming (DP) method which is similar to that of Alidaee and Kochenberger [10] to find an optimal solution. We assume each d_i to be integer, i.e., we use DP to solve a discretized version of the original problem (see [13,Chapter 4]).

We use a backward dynamic programming in which there are M stages. During each stage $i = M, M-1, M-2, \ldots, 1$, let $L \in \{0, 1, 2, \ldots, D\}$ denote the units of demand that will be satisfied by supplier $1, 2, \ldots, i-1$, and define $G_i(L)$ as the minimum total cost of satisfying the remaining $D - L$ units of demand to be satisfied by supplier $i, i+1, \ldots, M$.

The dynamic programming recursion is given by

$$G_i(L) = \min \left\{ G_{i+1}(L), \min_{1 \leqslant d_i \leqslant b_i} \left\{ c_i d_i + s_i n_i^* + \frac{h_i d_i^2}{2Dn_i^*} + F_i + G_{i+1}(L+d_i) \right\} \right\}, \tag{8}$$

where n_i^* is uniquely determined by d_i. We also need the boundary conditions $G_i(D)=0$ for $1 \leqslant i \leqslant M+1$, $G_i(L)=\infty$ for $D-L > \sum_{k=i}^{M} b_i$, and $G_{M+1}(L)=\infty$ for $0 \leqslant L < D$. The optimal solution can be obtained after $G_1(0)$ is calculated.

4.2. Improvement

The following properties can be used to improve the efficiency of the DP method, i.e., to get $G_i(L)$ directly without the calculation in (8).

Property 2. *If $G_{i+1}(L) \leqslant G_{i+1}(\gamma_i) + c_i + s_i + h_i/2D + F_i$, where $\gamma_i = \min\{L+b_i, D\}$, then $G_i(L) = G_{i+1}(L)$.*

Proof.

$$\begin{aligned} &\min_{d_i, n_i} \left\{ c_i d_i + s_i n_i + \frac{h_i d_i^2}{2Dn_i} + F_i + G_{i+1}(L+d_i) \right\} \\ &\quad \geqslant \min_{d_i, n_i} \left\{ c_i d_i + s_i n_i + \frac{h_i d_i^2}{2Dn_i} + F_i \right\} + \min_{d_i} \{ G_{i+1}(L+d_i) \} \\ &\quad \geqslant c_i + s_i + \frac{h_i}{2D} + F_i + G_{i+1}(\gamma_i). \end{aligned}$$

This yields

$$G_{i+1}(L) \leqslant \min_{d_i, n_i} \left\{ c_i d_i + s_i n_i + \frac{h_i d_i^2}{2Dn_i} + F_i + G_{i+1}(L+d_i) \right\}. \tag{9}$$

Combining the above inequality with (8) completes the proof. □

Property 3. *If there exists L^c such that $G_{i+1}(L^c) \leqslant c_i + s_i + h_i/2D + F_i < G_{i+1}(L^c - 1)$, then $G_i(L) = G_{i+1}(L)$ for any $L > L^c$.*

Proof. This follows from the non-increasing property of $G_i(L)$ in L. □

An example of the DP is shown in Appendix A. The time complexity of the DP is in $O(MDd_{\max})$, which is pseudopolynomial. As shown in Section 6.2, the DP may not be efficient when the total demand is very large or, when there are a significant number of suppliers to choose from. In the following section, we propose a heuristic algorithm based on local search to deal with large-scale problems.

5. Scatter search

Scatter search (see [14]) is a metaheuristic algorithm. A scatter search contains the following basic steps: construct a set of feasible solutions which is referred to as the reference set; combine the solutions in the reference set for creating new ones; use the new solutions to update the reference set. These steps are repeated until the number of iterations reaches a predetermined upper limit. We use the following notation to describe the algorithm:

P	a set of solutions which are used to construct a reference set
PSize	the size of **P**
RefSet	the reference set of solutions
RefSetSize	the size of **RefSet**
Subsets	a list of subsets which are pairs of solutions in **RefSet**
Pool	a set of solutions generated with the combination method
NewElements	a Boolean indicator about whether or not the reference set is updated
Iter	the number of iterations
MaxIter	the maximum number of iterations

5.1. Diversity generation method

A diversity generation method is used to generate a collection of initial solutions **P**. For **P**, we need to generate *PSize* distinct solutions, some of which might be infeasible. The purpose of the generation is to have initial solutions that are diverse in the solution space. In our design of the scatter search, we used a simple diversity generation method in which the d_i for each supplier i is generated arbitrarily between 0 and b_i.

5.2. Improvement method

In scatter search, an improvement method is used to transform the initial solutions into feasible and better solutions. The design of the improvement method is motivated by Property 1. We have shown in Section 3 that this property does not hold when the planning horizon is finite. One observation that can be inferred, however, is that fewer suppliers are preferred. Thus, during the process of the scatter search, we always try to reduce the number of suppliers.

For the improvement method, if $\sum_{i=1}^{M} d_i = D$ for a specific solution, then this solution is feasible, and we do nothing. Otherwise, first put the suppliers in a random sequence. If $\sum_{i=1}^{M} d_i > D$, we make $\sum_{i=1}^{M} d_i$ equal to D by reducing the ordering quantity of the first several suppliers as much as possible; if $\sum_{i=1}^{M} d_i < D$, we make $\sum_{i=1}^{M} d_i$ equal to D by increasing the ordering quantity of the first several suppliers as much as possible. The method is described as follows:

1. Let $D' = \sum_{i=1}^{M} d_i$. **if** $(D' = D)$, **then** stop. **else** randomize the sequence of the suppliers $1, 2, \ldots, M$.
   Let $\pi[j]$ denote the jth supplier in the new sequence.
2. **if** $(D' < D)$, **then goto** Step 3.
   for $(j = 1, 2, \ldots, M)$
      if $(d_{\pi[j]} < D' - D)$, **then** $D' = D' - d_{\pi[j]}$, $d_{\pi[j]} = 0$.
      else $d_{\pi[j]} = d_{\pi[j]} - D' + D$, stop.
   end for
3. **for** $(i = 1, 2, \ldots, M)$
      if $(b_{\pi[j]} - d_{\pi[j]} < D - D')$, **then** $D' = D' + b_{\pi[j]} - d_{\pi[j]}$, $d_{\pi[j]} = b_{\pi[j]}$.
      else $d_{\pi[j]} = d_{\pi[j]} + D - D'$, stop.
   end for

5.3. Reference set update method

To ensure that the quality and diversity of the solution pool can be improved after each iteration, the reference set needs to be updated. A reference set update method is for generating and updating **RefSet** consisting of *RefSetSize* "best"(either in quality or diversity) solutions in **P**.

In our design, half of the reference set is obtained from the best $RefSetSize/2$ solutions in **P**, and the other half is composed of the $RefSetSize/2$ solutions in **P-RefSet** with the maximal minimum Euclidean distances. Here, we define the Euclidean distances between two solutions $\mathbf{x} = \{x_1, x_2, \ldots, x_M\}$ and $\mathbf{y} = \{y_1, y_2, \ldots, y_M\}$ to be

$$ED(\mathbf{x}, \mathbf{y}) = \sqrt{\sum_{i=1}^{M}(x_i - y_i)^2}. \tag{10}$$

For $\mathbf{A}$, a set of M-dimensional solutions, the minimum Euclidean distance for a solution $\mathbf{x}$ in terms of $\mathbf{A}$ is defined to be

$$ED_{\min}(\mathbf{x}, \mathbf{A}) = \min_{\mathbf{y}\in\mathbf{A}}\{ED(\mathbf{x}, \mathbf{y})\}. \tag{11}$$

5.4. Subset generation method

To generate new solutions using the current reference set, we first combine the *RefSetSize* solutions in the reference set to generate a number of subsets. We only consider the generation of subsets of size two, i.e., choosing two different solutions to form a subset. The subset generation method can produce a maximum of $RefSetSize \cdot (RefSetSize - 1)/2$ subsets. To guarantee the quality of the subsets, pairs of reference solutions that have already been combined before will not be used to combine again, i.e., we only consider the solutions in the reference set which were not combined in previous iterations. Hence the number of subsets generated in each iteration might be less than the maximum.

5.5. Combination method

A solution combination method is used to create more feasible solutions from a given subset of solutions generated in the subset generation method. We use a simple linear combination method. For each pair of reference solutions $\{\mathbf{d}', \mathbf{d}''\}$, we can obtain three new solutions $\hat{\mathbf{d}}_1$, $\hat{\mathbf{d}}_2$, and $\hat{\mathbf{d}}_3$:

$$\hat{\mathbf{d}}_1 = \frac{2+r}{2}\mathbf{d}' - \frac{r}{2}\mathbf{d}'', \tag{12}$$

$$\hat{\mathbf{d}}_2 = \frac{2-r}{2}\mathbf{d}' + \frac{r}{2}\mathbf{d}'', \tag{13}$$

$$\hat{\mathbf{d}}_3 = -\frac{r}{2}\mathbf{d}' + \frac{2+r}{2}\mathbf{d}'', \tag{14}$$

where r is a random number between 0 and 1. The three new solutions, after being improved to feasible solutions, are then appended to **Pool**.

Though this method is very simple, it is extremely suitable for the solution of our model because of the property of the supplier profile $\mathbf{d} : \sum_{i=1}^{M} d_i = D$ and $0 \leqslant d_i \leqslant b_i$. Under the above linear combination approach, $\hat{\mathbf{d}}_2$ retains such a property, so it is automatically a feasible solution.

5.6. Description of the algorithm

Combining the five parts described in Sections 5.1–5.5, the whole scatter search method is summarized as follows.

1. Initialize $\mathbf{P} = \emptyset$. Construct $\mathbf{d}$ by generating d_i randomly in $[0, b_i]$, for $i = 1, 2, \ldots, M$. Transform $\mathbf{d}$ to $\bar{\mathbf{d}}$ by the improvement method. Let $\mathbf{P} = \mathbf{P} \cup \{\bar{\mathbf{d}}\}$. Repeat until the size of $\mathbf{P}$ reaches *PSize*.
2. Sort the solutions in $\mathbf{P}$ in nondecreasing order of the objective values. Append the first $Ref\,SetSize/2$ solutions to **Ref Set** and delete them from $\mathbf{P}$. Let $Iter = 1$.
3. For each $\mathbf{d}$ in **P-RefSet** and $\mathbf{y}$ in **Ref Set**, calculate $ED(\mathbf{d}, \mathbf{y})$. Select the solution $\mathbf{d}' \in$ **P-Ref Set** that maximize $ED_{\min}(\mathbf{d}, \mathbf{Ref\,Set})$. Append $\mathbf{d}'$ to **Ref Set** and remove it from $\mathbf{P}$. Repeat this step for $Ref\,SetSize/2$ times. Let $NewElements =$ TRUE.
4. **while** ($NewElements =$ TRUE) **do**
   (a) Generate **Subsets** with the subset generation method. Let $NewElements =$ FALSE.

(b) **while** (**Subsets** $\neq \emptyset$) **do**
  i. Select the next subset **s** in **Subsets**.
  ii. Apply the combination method to **s** to obtain three trial solutions $\hat{\mathbf{d}}_1$, $\hat{\mathbf{d}}_2$,and $\hat{\mathbf{d}}_3$. Apply the improvement method to them to get three feasible solutions $\bar{\mathbf{d}}_1$, $\bar{\mathbf{d}}_2$, and $\bar{\mathbf{d}}_3$.
  iii. Append $\bar{\mathbf{d}}_1$, $\bar{\mathbf{d}}_2$, and $\bar{\mathbf{d}}_3$ to **Pool** and delete **s** from **Subsets**.
  end while
(c) Update **Ref Set** by selecting the best *Ref SetSize* solutions in **Ref Set** $\cup$ **Pool**. **if** there is at least one new solution in the updated reference set, **then** let *NewElements* = TRUE.

end while

5. **if** (*Iter* < *MaxIter*) **then** delete the last *RefSetSize*/2 solutions from **Ref Set**, build a new set **P** using the diversification generation method and improvement method, let *Iter* = *Iter* + 1, **goto** Step 3.

6. Computational results

In this section, we study the benefits of our optimal solution subject to the RHH solution with respect to cost differences, the running time of DP, and the performance of the scatter search algorithm.

6.1. Cost savings

6.1.1. Experimental design

To generate problem instances, the parameters of the candidate suppliers were randomly generated from the following uniform distributions: $c_i \sim [c_{\min}, c_{\max}]$, $s_i \sim [s_{\min}, s_{\max}]$, $F_i \sim [F_{\min}, F_{\max}]$, $b_i \sim [b_{\min}, b_{\max}]$ for $i = 1, 2, \ldots, M$. In the default setting, we have a planning horizon of one year, $c_{\min} = 1.0$, $c_{\max} = 2.0$, $s_{\min} = 60$, $s_{\max} = 200$, $F_{\min} = 20$, $F_{\max} = 200$, $b_{\min} = 100$, and $b_{\max} = 400$. Also, let the holding cost $h_i = 1.0$, and the total demand $D = 1000$.

For each group of parameters, we randomly generated 100 instances, and solved each by the DP algorithm and the RHH method, respectively. Define

$$RE_{\text{RHH}} = \frac{TC_{\text{RHH}} - TC_{\text{DP}}}{TC_{\text{DP}}}$$

as the relative error of the RHH solution subject to the optimal solution, where TC_{RHH} is the total cost of the solution obtained through the RHH approximation scheme, and TC_{DP} is the total cost of the DP solution. We will report MRE_{RHH}, the mean of the RE_{RHH} over the 100 problem instances. We are able to observe how the cost differences are affected in the following three cases.

6.1.2. Demand changes

We used the parameters as the default setting except that $D = r\sum_{i=1}^{M} b_i$, where $r \in \{0.1, 0.2, \ldots, 1.0\}$ indicates the slackness of the problem with respect to the ratio of demand over total supplying capacity. When $r = 1.0$, all capacity has to be used to satisfy the demand, and a trivial optimal solution is to use all suppliers. In this case, the RHH solution is always the same as the optimal solution. When $r < 1.0$, not all suppliers are used, and the RHH solution may cause greater costs. In Fig. 1, we report the MRE_{RHH} for different r values and different numbers of suppliers $M = 10$, 15, and 20, respectively.

We can see from Fig. 1 that the smaller the ratio of demand over total capacity, the greater are the average differences between the RHH solution and the optimal solution. This is not surprising in that a smaller r implies fewer suppliers are needed to satisfy the demand, or greater possibility for RHH method to get the suppliers different from the optimal ones, and therefore results in a relatively worse performance of the approximation scheme. Consider a manufacturer who faces a significant number of suppliers. Even though the capacity of each supplier is small, the total capacity may be rather large. In this case, our model can generate much better results than the RHH method.

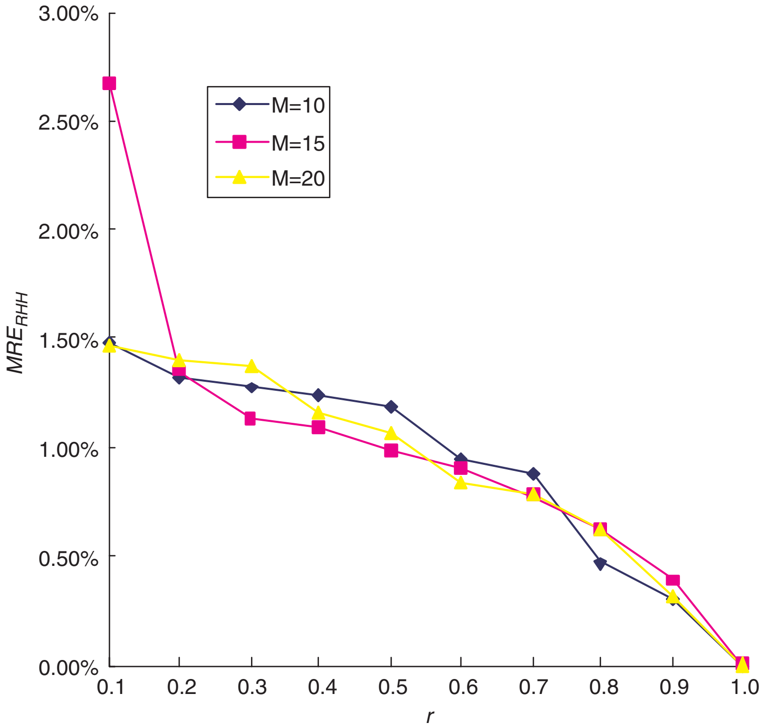

Fig. 1. MRE_{RHH} for various ratios of demand over total capacity.

6.1.3. Different administration fees

Administration fees play an important role in distinguishing one supplier from another. We generated problem instances with the default parameters except that $D = 0.3.\sum_{i=1}^{M} b_i$, $F_{\max}$ is set to be $5F_{\min}$, and $F_{\min} = 10, 20, \ldots, 110$. The trend of MRE_{RHH} in terms of $F_{\min}$ is shown in Fig. 2 for different numbers of suppliers.

All other conditions being the same, we see from Fig. 2 that as the administration costs are decreased, our model shows much better performance than the RHH solution. Given the fact that manufacturers are trying to reduce administration costs with their suppliers, our model can result in much greater cost savings.

6.1.4. Different planning horizons

We now show the performance of our solution when the planning horizon changes. All other parameters being the same as in the default settings, we solved problems with different planning horizons ranging from one month to one year. We calculated MRE_{RHH} for each planning horizon, and the results are shown in Fig. 3. It is evident that the shorter the length of the planning horizon, the larger the difference between the RHH solution and the exact optimal solution.

Given the increasingly dynamic business environment, it is far from realistic to assume that a manufacturer will sign an outsourcing contract with an infinite planning horizon. Rather, it is quite common for a manufacturer to make a particular outsourcing decision for a short period, say several months. In this case, our computational results show that a considerable cost savings could be achieved through calculating the exact optimal solution. This result clearly reveals the importance of dealing with the finite-time model directly.

6.2. Running time of DP

The parameters for each supplier were randomly generated under the default setting in Section 6.1.1 except that $b_i \sim [0.5\bar{b}, 1.5\bar{b}]$ for $i = 1, 2, \ldots, M$, and $D = 0.3.\sum_{i=1}^{M} b_i$, where $\bar{b}$ is the mean capacity of a single supplier. For different combinations of $\bar{b}$ and M, the average running time is shown in Table 4. The computer in the experiment has

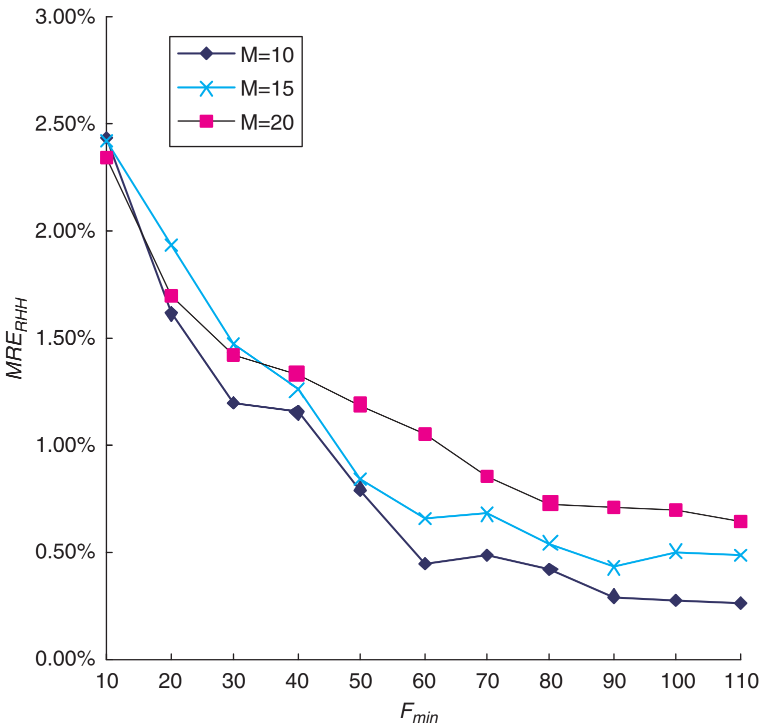

Fig. 2. MRE_{RHH} for various minimum administration fees.

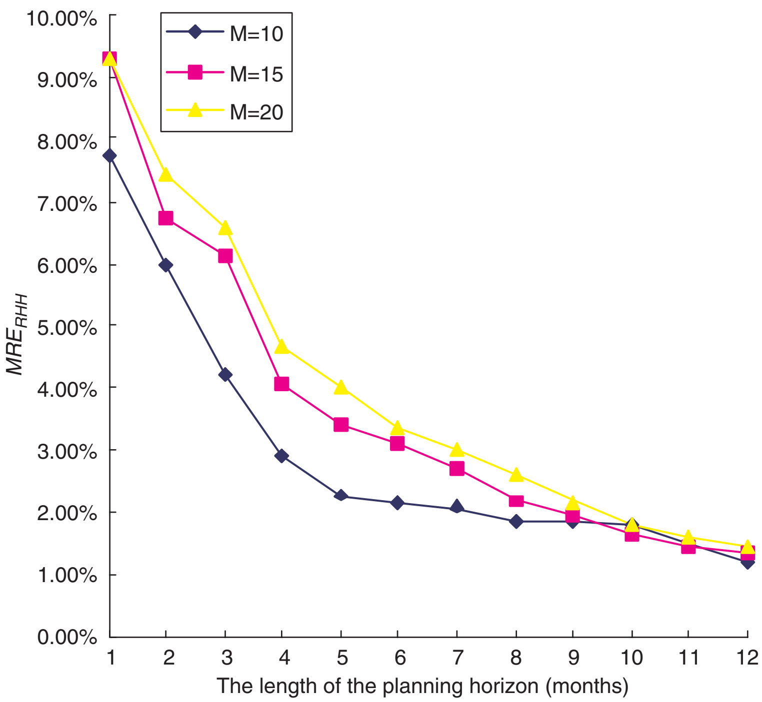

Fig. 3. MRE_{RHH} for different lengths of the planning horizon.

Table 4
The mean running time (s) of DP

M	$\bar{b}$												
	10	20	50	100	200	300	400	500	600	700	800	900	1000
10	0.0	0.0	0.1	0.3	1.4	3.3	6.0	9.4	13.8	18.5	26.6	30.8	38.5
20	0.0	0.1	0.8	3.5	14.3	32.9	61.2	103.0	151.1	202.0	304.6	383.0	456.7
40	0.4	1.4	9.9	45.4	259.8	608.0	1113.1	1568.9	2930.0	3083.1	3512.7	–	–
80	5.7	24.7	193.9	895.4	3574.6	–	–	–	–	–	–	–	–

"–" indicates that the mean running time is over 3600 s.

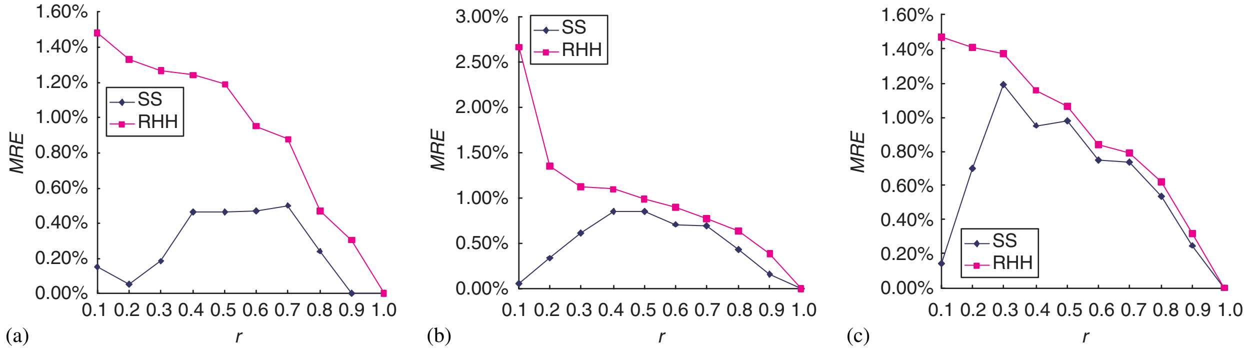

Fig. 4. MRE_{SS} and MRE_{RHH} for (a) $M = 10$, (b) $M = 15$, and (c) $M = 20$.

a CPU of Intel Pentium III, 666 MHz, 256 MB RAM. The test results show that it is time consuming to use DP to solve the problem when the size of the problem becomes very large.

6.3. Performance of scatter search

To evaluate the performance of the scatter search algorithm, we compared the scatter search solution with both the optimal DP solution and the RHH solution. The parameters of the candidate suppliers were uniformly randomly generated within a predetermined range, which is the same as the default setting in Section 6.1.1 except that $D = r\sum_{i=1}^{M} b_i$. Let $r = 0.1, 0.2, \ldots, 1.0$, and $M = 10$, 15, and 20. For each group of parameters, we generated 100 instances and solved each of them by the RHH, SS (scatter search), and DP, respectively. Define MRE_{SS} as the mean relative error of SS solution subject to the optimal solution obtained with DP. We calculated MRE_{RHH} and MRE_{SS}. The results are reported in Figs. 4 (a), (b), and (c) for different numbers of suppliers.

In all three figures, the scatter search, though not able to guarantee optimality, generally resulted in a better solution than the RHH one. When the demand rate is very large, it is time consuming to obtain the exact optimal solution with the dynamic program. In this case, the scatter search provides an efficient and effective approach to obtain a satisfactory solution.

7. Further discussion

How to handle multiple capacitated suppliers is an interesting problem, especially for a manufacturer which outsources its production from many smaller overseas suppliers. In addition to the existing literature and our paper, more sophisticated models are desired to address this issue.

One direction is to study the problem under different demand settings. We study an EOQ-type continuous-time deterministic model in this paper. It would be interesting to explore a corresponding discrete-time case, i.e., a dynamic lot sizing model with multiple capacitated suppliers. Another possibility is to develop the structure of the optimal acquisition policy when the demand rate becomes stochastic.

The limitation of capacity contributes much to the difficulty of our problem solving, and affects the level of reducing inventory costs. In our model, the capacity of each supplier is fixed, which can be interpreted as the maximum production output of a supplier. In some cases, however, the capacity of a supplier is (partly) determined by the contract with its buyer. From the manufacturer's point of view, extra cost savings are expected because the capacity becomes a new decision variable, which makes the problem more compelling.

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Appendix A. An example of the DP algorithm

Consider the supplier-selection problem with a total demand of D=5, and seven candidate suppliers whose parameters are shown in Table 5. There are seven stages in the DP, and we start with Stage 7.

Based on the definition of $G_i(L)$, we know that

$$G_i(5) = 0 \quad \text{for each } i = 7, 6, \ldots, 1, 0.$$

At Stage 7, $G_7(L)$ is calculated as follows for $L = 0, 1, \ldots, 5$:

$$G_7(L) = \infty \quad \text{for } L = 0, 1,$$
$$G_7(2) = c_7 \cdot 3 + s_7 \cdot n_7^* + \frac{h_7 \cdot 3^2}{2 \cdot D \cdot n_7^*} + F_7 = 35.2,$$
$$G_7(3) = c_7 \cdot 2 + s_7 \cdot n_7^* + \frac{h_7 \cdot 2^2}{2 \cdot D \cdot n_7^*} + F_7 = 33.6,$$
$$G_7(4) = c_7 \cdot 1 + s_7 \cdot n_7^* + \frac{h_7 \cdot 1^2}{2 \cdot D \cdot n_7^*} + F_7 = 32.2.$$

At Stage 6:

$$G_6(0) = \infty,$$

$$G_6(1) = c_6 \cdot 1 + s_6 \cdot 1 + \frac{h_6 \cdot 1^2}{2 \cdot D \cdot 1} + F_6 + G_7(2) = 60.0,$$

Table 5
Parameters of suppliers

Supplier No. i	c_i	s_i	F_i	h_i	b_i
1	1.7	5.0	10.0	1.0	2
2	1.6	13.0	11.0	1.0	3
3	1.3	5.0	16.0	1.0	2
4	1.1	8.0	29.0	1.0	1
5	1.2	9.0	18.0	1.0	2
6	1.7	10.0	12.0	1.0	1
7	1.1	14.0	17.0	1.0	3

Table 6
The calculation of $G_i(L)$

i	L				
	0	1	2	3	4
5	65.0	60.0	35.2	29.8	23.8
4	65.0	60.0	35.2	29.8	23.8
3	59.2	53.8	35.2	24.0	22.4
2	53.7	51.6	29.7	24.0	22.4
1	48.5	42.8	29.7	18.8	16.8

$$G_6(2) = \min_{d=1,2}\left\{c_6\cdot d + s_6\cdot 1 + \frac{h_6\cdot d^2}{2\cdot D\cdot 1} + F_6 + G_7(2+d)\right\},$$
$$= \min\left\{G_7(2), c_6\cdot 1 + s_6\cdot 1 + \frac{h_6\cdot 1^2}{2\cdot D\cdot 1} + F_6 + G_7(3)\right\} = G_7(2) = 35.2,$$

$$G_6(3) = \min\left\{G_7(3), c_6\cdot 1 + s_6\cdot 1 + \frac{h_6\cdot 1^2}{2\cdot D\cdot 1} + F_6 + G_7(4)\right\} = G_7(3) = 33.6,$$

$$G_6(4) = \min\left\{G_7(4), c_6\cdot 1 + s_6\cdot 1 + \frac{h_6\cdot 1^2}{2\cdot D\cdot 1} + F_6 + G_7(5)\right\}$$
$$= c_6\cdot 1 + s_6\cdot 1 + \frac{h_6\cdot 1^2}{2\cdot D\cdot 1} + F_6 + G_7(5) = 23.8.$$

We then calculate $G_i(L)$, $i = 5, 4, \ldots, 1$, $L = 0, 1, \ldots, 4$ in a similar way. The values of these $G_i(L)$'s are listed in Table 6. The minimum total cost obtained by using DP is $G_1(0) = 48.5$. Back tracking yields an optimal solution $\mathbf{d}^* = \{2, 3, 0, 0, 0, 0, 0\}$.

Appendix B. An example of the scatter search algorithm

We use the same supplier-selection problem as in Appendix A to show the details of several key steps in the scatter search algorithm. For demonstration purpose, let $PSize = 6$ and $Ref\,Size = 4$.

Step 1: Initialize $\mathbf{P} = \{0, 0, 0, 0, 0, 0, 0\}$. Using the diversification method, we get $\mathbf{d} = \{1, 2, 0, 1, 1, 0, 1\}$, which is improved to $\bar{\mathbf{d}} = \{1, 2, 0, 0, 1, 0, 1\}$. $\mathbf{P} = \mathbf{P} \cup \{\bar{\mathbf{d}}\} = \{\{1, 2, 0, 0, 1, 0, 1\}\}$. Repeat until there are *PSize* solutions in $\mathbf{P}$. We finally have $\mathbf{P} = \{\{1, 2, 0, 0, 1, 0, 1\}, \{0, 1, 1, 0, 0, 1, 2\}, \{0, 3, 0, 0, 2, 0, 0\}, \{2, 0, 0, 0, 1, 0, 2\}, \{0, 0, 0, 1, 0, 1, 3\}, \{1, 2, 0, 0, 2, 0, 0\}\}$.

Step 2: Sorting the solutions in $\mathbf{P}$ in non-decreasing order of the objective function values yields $\mathbf{P} = \{\{0, 3, 0, 0, 2, 0, 0\}, \{1, 2, 0, 0, 2, 0, 0\}, \{2, 0, 0, 0, 1, 0, 2\}, \{0, 0, 0, 1, 0, 1, 3\}, \{1, 2, 0, 0, 1, 0, 1\}, \{0, 1, 1, 0, 0, 1, 2\}\}$. We add the first two solutions in $\mathbf{P}$ to **Ref Set** and delete them from $\mathbf{P}$ at the same time. Hence we have **Ref Set** $= \{\{0, 3, 0, 0, 2, 0, 0\}, \{1, 2, 0, 0, 2, 0, 0\}\}$, and $\mathbf{P} = \{\{2, 0, 0, 0, 1, 0, 2\}, \{0, 0, 0, 1, 0, 1, 3\}, \{1, 2, 0, 0, 1, 0, 1\}, \{0, 1, 1, 0, 0, 1, 2\}\}$.

Step 3: Select two solutions in **P-Ref Set** that have the largest minimum Euclidean distance. Firstly, we calculate the minimum Euclidean distances of the four solutions in **P-Ref Set** as follows:

$$ED_{\min}(\{2, 0, 0, 0, 1, 0, 2\}, \mathbf{Ref\,Set}) = \min\left\{\sqrt{18}, \sqrt{10}\right\} = \sqrt{10},$$
$$ED_{\min}(\{0, 0, 0, 1, 0, 1, 3\}, \mathbf{Ref\,Set}) = \min\left\{\sqrt{24}, \sqrt{20}\right\} = \sqrt{20},$$
$$ED_{\min}(\{1, 2, 0, 0, 1, 0, 1\}, \mathbf{Ref\,Set}) = \min\left\{\sqrt{4}, \sqrt{2}\right\} = \sqrt{2},$$
$$ED_{\min}(\{0, 1, 1, 0, 0, 1, 2\}, \mathbf{Ref\,Set}) = \min\left\{\sqrt{14}, \sqrt{12}\right\} = \sqrt{12}.$$

Since $\sqrt{20}$ is the largest, we append $\{0, 0, 0, 1, 0, 1, 3\}$ to **Ref Set**, and delete it from $\mathbf{P}$.

Then we repeat the calculation of the Euclidean distances of the solutions in **P-Ref Set** with the updated **Ref Set**:

$$ED_{\min}(\{2,0,0,0,1,0,2\}, \textbf{Ref Set}) = \min\left\{\sqrt{18}, \sqrt{10}, \sqrt{8}\right\} = \sqrt{8},$$
$$ED_{\min}(\{1,2,0,0,1,0,1\}, \textbf{Ref Set}) = \min\left\{\sqrt{4}, \sqrt{2}, \sqrt{12}\right\} = \sqrt{2},$$
$$ED_{\min}(\{0,1,1,0,0,1,2\}, \textbf{Ref Set}) = \min\left\{\sqrt{14}, \sqrt{12}, \sqrt{4}\right\} = \sqrt{4}.$$

So we append $\{2,0,0,0,1,0,2\}$ to **Ref Set** and remove it from **P**, for $\sqrt{8} > \sqrt{4} > \sqrt{2}$. Now the updated **Ref Set** $= \{\{0,3,0,0,2,0,0\}, \{1,2,0,0,2,0,0\}, \{0,0,0,1,0,1,3\}, \{2,0,0,0,1,0,2\}\}$.

Step 4: Subset generation method. Append the following pairs of solutions to **Subsets**:

$$\{\{0,3,0,0,2,0,0\}, \{1,2,0,0,2,0,0\}\}, \{\{0,3,0,0,2,0,0\}, \{0,0,0,1,0,1,3\}\},$$
$$\{\{0,3,0,0,2,0,0\}, \{2,0,0,0,1,0,2\}\}, \{\{1,2,0,0,2,0,0\}, \{0,0,0,1,0,1,3\}\},$$
$$\{\{1,2,0,0,2,0,0\}, \{2,0,0,0,1,0,2\}\}, \{\{0,0,0,1,0,1,3\}, \{2,0,0,0,1,0,2\}\}.$$

Step 5: Apply the combination method to obtain trial solutions. For example, for the pair $\{\{0,3,0,0,2,0,0\}, \{2,0,0,0,1,0,2\}\}$, when the random number $r = 0.459$, we have the following three trial solutions:

$$\begin{aligned}\hat{\mathbf{d}}_1 &= \frac{2+0.459}{2}\{0,3,0,0,2,0,0\} - \frac{0.459}{2}\{2,0,0,0,1,0,2\}\\ &= \{-0.2295, 3.2295, 0, 0, 2.2295, 0, -0.2295\},\end{aligned}$$

$$\begin{aligned}\hat{\mathbf{d}}_2 &= \frac{2-0.459}{2}\{0,3,0,0,2,0,0\} + \frac{0.459}{2}\{2,0,0,0,1,0,2\}\\ &= \{0.459, 2.3115, 0, 0, 1.7705, 0, 0.459\},\end{aligned}$$

$$\begin{aligned}\hat{\mathbf{d}}_3 &= -\frac{0.459}{2}\{0,3,0,0,2,0,0\} + \frac{2+0.459}{2}\{2,0,0,0,1,0,2\}\\ &= \{2.459, -0.6885, 0, 0, 0.7705, 0, 2.459\}.\end{aligned}$$

Rounding the three trial solutions and applying the improvement method to them, we get three new solutions $\{0,3,0,0,2,0,0\}$, $\{0,2,0,0,2,0,1\}$, and $\{2,0,0,0,1,0,2\}$ and append them to **Pool**. We also delete the pair $\{\{0,3,0,0,2,0,0\}, \{2,0,0,0,1,0,2\}\}$ from **Subsets**. If **Subsets** $\neq \emptyset$, repeat doing Step 5.

Step 6: Update **Ref Set** by appending the best four solutions in **Ref Set** $\cup$ **Pool**. If there is a new solution in **Ref Set**, goto Step 5. Otherwise, we delete the last two solutions in **Ref Set**, use the same procedures as that of Step 1 to build a new **P**, and goto Step 2.

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