Jumping the Line, Charitably: Analysis and Remedy of Donor-Priority Rule

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Abstract. The ongoing shortage of organs for transplantation has generated an expanding literature on efficient and equitable *allocation* of the donated cadaveric organs. By contrast, organ donation has been little explored. In this paper, we develop a parsimonious model of organ donation to analyze the welfare consequences of introducing the donor-priority rule, which grants registered organ donors priority in receiving organs should they need transplants in the future. We model an individual's decision to join the donor registry, which entails a trade-off between abundance of supply, exclusivity of priority, and cost of donating (e.g., psychological burden). Assuming heterogeneity in the cost of donating only, we find the introduction of the donor-priority rule leads to improved social welfare. By incorporating heterogeneity in the likelihood of requiring an organ transplant and in organ quality, we show that, in contrast to the literature, introducing the donor-priority rule can lower social welfare because of unbalanced incentives across different types of individuals. In view of the potentially undesirable social-welfare consequences, we consider a freeze-period remedy, under which an individual is not entitled to a higher queueing priority until after having been on the organ-donor registry for a specified period of time. We show this additional market friction helps rebalance the incentive structure, and in conjunction with the donor-priority rule, can guarantee an increase in social welfare by boosting organ supply without compromising organ quality or inducing excessively high costs of donating.

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1. Introduction

The United States, as with much of the rest of the world, is experiencing an organ-shortage crisis with about 18 people per day dying while waiting for transplants and a new candidate being added to the transplant wait list every 10 minutes (Organdonor.gov 2016). In the decade leading up to 2014, the number of people wait-listed for organ transplants increased by 3.5 times, but the number of people who pledged to donate their organs following death grew by merely 1.7 times. Because cadaveric organs remain a major source of organs for transplantation, the organ-shortage crisis can be largely attributed to a low share of registered organ donors. The shortage is particularly alarming in populated states, such as Texas, New York, and California, where only 7%, 15%, and 28%, respectively, of adults are registered donors (Donate Life America 2011).

Numerous initiatives have been proposed to encourage more people to add their names to the organ-donor

registry. The contemporary discourse focuses on educational measures to enhance public awareness of the benefits of organ donation. In May 2012, Facebook unveiled a new sharing function that enables its users to advertise their donor status on their timelines. In July 2016, Apple Inc. announced that iPhone users could become nationally registered donors using the Health app. In addition, the United States and United Kingdom have experimented with "nudge" strategies to encourage minority ethnic groups to become organ donors (Morgan et al. 2015).

Additionally, two noteworthy policy initiatives have been proposed: (i) The first is the donor-priority rule, which provides priority status to individuals registering to become potential organ donors. Under the rule, should registered donors need organ transplants, they are given priority over nondonors in receiving cadaveric organs. The introduction of the donor-priority rule in Israel in April 2012 has led to a significant increase in the registration rate (Stoler et al. 2017). (ii) The second is the presumed-consent (a.k.a. "opt-out") policy, which, in contrast to the current U.S. practice, automatically registers adults as organ donors unless they opt out of the organ-donation program. Various studies have endorsed legislation of the presumed-consent policy (see, e.g., Abadie and Gay 2006), but it faces numerous hurdles, including the public's fear of misrepresentation of individuals' willingness to donate (Johnson and Goldstein 2003, Teresi 2012).

Our paper focuses on analyzing the donor-priority rule, which has been implemented in Chile, Israel, and Singapore. In Chile and Singapore, it has been implemented along with the opt-out policy such that (1) all individuals are, by default, registered organ donors and belong to the same priority class, and (2) those who opt out have a downgraded priority. Israel, like the United States, has adopted the opt-in policy but has implemented the donor-priority rule since April 1, 2012, which has resulted in a significant increase in registration rates (Stoler et al. 2017). The donor-priority rule, as with other policy proposals, is not without ethical issues. One ethical issue associated with the donor-priority rule is that it bases allocation priority "on organ donation rather than medical need" (Goldberg and Trotter 2016, p. 2513). Another ethical issue is the next of kin might veto cadaveric donation of a deceased individual; relevant to this issue, Lavee and Brock (2012, p. 709) observe from the Israeli practice that "families of deceased persons with a donor card have traditionally approved organ donation and almost never vetoed donation as they consider the deceased's signature on the donor card as a signed will."

As a U.S.-based private experiment, in 2002, a former insurance broker founded LifeSharers, a nonprofit network of organ donors who pledge to donate their organs to other members first in need in the event of premature death. LifeSharers attracted more than 12,000 members before it shut down in March 2016 without facilitating any organ transplantation. When reflecting on the failure of LifeSharers, Roth (2013, p. 8) notes that a national priority system has a substantial number of donors even without priority access (these donors would not exist in a private members-only club): "Under a priority system, priority access to those donors' organs would be the incentive for additional donation decisions. ... This is what makes a national priority system a more feasible system than a private members-only club." In other words, the failure of LifeSharers does not set a precedent for a national policy; rather, it underscores the necessity of a national policy initiative for the donor-priority rule to work because of the existence of a large altruistic base.

We develop a queueing theoretic model of donor registration and organ allocation, allowing the interaction between demand priority and endogenous supply.¹ In modeling the trade-offs behind each individual's decision to register to become a potential organ donor, we follow Kessler and Roth's (2012) approach by assuming each individual incurs a *cost of donating* associated with registering as an organ donor, which may be either positive or negative; a positive cost of donating represents an internal loss (e.g., fear and discomfort) from becoming a registered organ donor, whereas a negative cost of donating represents an internal reward. Different from Kessler and Roth (2012), we capture each individual's utility from organ transplantation using the qualityadjusted life expectancy (QALE) and applying the approximation results from the queueing literature (e.g., Zenios 1999).

A commonly felt concern about the donor-priority rule is that it may be perceived as "oppressive" in that it may induce some individuals to register against their own low willingness to donate. In other words, this concern means the priority stemming from becoming a registered organ donor can be so valuable that certain individuals may opt to register to become organ donors despite their excessively high costs of donating. Accounting for the cost of donating makes the effect of the donor-priority rule not immediately clear. Our analysis helps elucidate the social-welfare consequences of the donor-priority rule under a variety of settings and policy environments.

First, in a benchmark in which individuals are heterogeneous in their costs of donating only, we show the introduction of the donor-priority rule will expand the size of the donor registry, increase the overall availability of obtaining an organ, and unequivocally result in increased social welfare. This result is consistent with Kessler and Roth's (2012) main finding.

Second, when the individuals are heterogeneous in health status as well as in their costs of donating, we show that, different from what Kessler and Roth (2012) would predict, the introduction of the donorpriority rule may indeed *lower* social welfare. The intuition is that under this rule, even individuals with the same cost of donating may respond differently in their decisions to register because they have different likelihoods of requiring organ transplants in the future. Specifically, we show that, ceteris paribus, the donor-priority rule, by providing a stronger incentive to high-risk individuals (i.e., those with a high likelihood of requiring organ transplants in the future) than to low-risk individuals, results in (1) a pool of organs with an average quality lower than that of the overall population and (2) a proportion of new organ donors with excessively high costs of donating. When this incentive structure becomes sufficiently asymmetric, the resultant social-welfare loss-resulting from the reduction in the average organ quality and the increase in the aggregate costs of donating—can outweigh the social-welfare gain from the expanded organ-donor registry. Furthermore, even when organ quality is homogeneous and the average organ quality does not decrease after introducing the donorpriority rule, the donor-priority rule can result in lower social welfare because it attracts those high-risk individuals with high costs of donating. Those highrisk individuals with excessively high costs of donating are "pressured" into registering because of their high likelihood of needing transplants and the slim chance of receiving an organ transplant if they remain unregistered. In other words, under the donor-priority rule, some individuals' decisions to register to become donors are individually rational but not collectively optimal.

Last and perhaps most interesting, mirroring the Israeli practice, we consider a freeze-period remedy, under which an individual is not entitled to a higher queueing priority until after having been on the organdonor registry for a specified period of time, which we refer to as a "freeze period." We prove the freezeperiod remedy, if well calibrated, can overcome the aforementioned effect of quality distortion under the donor-priority rule: when the remedy is imposed in conjunction with the donor-priority rule, the average quality of the donated organs can be restored to the level of the population average. The intuition behind this improvement is that the freeze-period remedy adds a friction to the organ-donation system, which provides a disincentive to *all* individuals for becoming organ donors. The strength of the disincentive differs across risk types such that high-risk individuals are discouraged to a greater extent than low-risk ones. Thus, a well-calibrated freeze-period remedy helps counteract the distorted incentive structure arising under the donor-priority rule. We analytically prove that the optimal freeze-period remedy, when implemented alongside the donor-priority rule, guarantees higher social welfare than before the introduction of the donor-priority rule: the remedy helps improves social welfare through boosting the supply of organs without severely compromising the quality or inducing excessively high costs of donating.

The rest of the paper is organized as follows. In Section 2, we review the relevant literature and highlight our main contributions. In Section 3, we describe our general modeling framework. In Section 4, we consider a benchmark in which all individuals differ only in their costs of donation. Building on that benchmark, Section 5 incorporates population heterogeneity in health status. In Section 6, we consider a freeze-period remedy. In Section 7, we use numerical experiments to illustrate our main findings. In Section 8, we analyze three extensions: on candidate autonomy, moral hazard, and dynamic registering decisions, respectively. In Section 9, we conclude the paper. All technical proofs are in the appendices. The online appendix provides technical details of several extensions of our main model.

2. Literature

Our paper contributes to a growing body of (broadly defined) operations management literature on organtransplant services, most of which focuses on optimal organ-allocation schemes (see, e.g., Su and Zenios 2004, 2006; Kong et al. 2010; Akan et al. 2012; Bertsimas et al. 2013; Sandikçi et al. 2013; Gentry et al. 2015; and Ata et al. 2017) and individual transplant surgeons' decisions (e.g., Alagoz et al. 2004, Howard 2002, and Zhang 2010); see section 2 of Dai and Tayur (2019) for an overview of the U.S. organ transplantation system, and Ata et al. (2018) for a review of this literature. To the best of our knowledge, our paper presents the first analytical model in the field of operations management to examine an organdonation policy. Our paper is relevant to several papers examining the queueing discipline for organ allocation. Su and Zenios (2004) develop and analyze a queueing model to study the role of patient choice in the kidney-transplant waiting system and highlight the conflict between equity and efficiency in kidney allocation. Our paper, with a specific focus on organ donation, also considers a priority-queueing discipline, but the queueing priority is principally tied to each individual's donor status. Su and Zenios (2006) propose an organ-allocation method whereby heterogeneous patients have to declare which types of kidneys they are willing to accept at the time they join the wait list (rather than at the time they are offered the kidney) and show this scheme eliminates the need for a lengthy search at the time of the kidney transportation to the transplantation site. Our analysis builds on the relationship between individuals' heterogeneous risk types and their decisions to join the donor registry; these decisions collectively determine the quality of donated organs. Our paper adopts a fluid-approximation approach developed by Zenios (1999), which provides a general representation of the transplantation waiting list with reneging because of patient death. Different from Zenios (1999), we consider both organ donation and organ transplantation with an emphasis on the former.

The economics literature has examined the practice of charitable fund-raising and giving; see, for example, Andreoni (1989), Eckel and Grossman (2003), and Landry et al. (2006). Yet theoretic modeling efforts on the U.S. organ-donation system under the donorpriority rule remains scant. The only paper that analytically models organ donation is by Kessler and Roth (2012). They develop an analytic model of individuals' decisions to become organ donors by weighing their cost of donating—a psychological friction in the organ-donation system—as well as their utility from gaining priority access associated with donor status. Our work differs from theirs mainly in two aspects. First, Kessler and Roth (2012) address the impact of the donor-priority rule mainly through behavioral experiments. Second, their analytical model, built to illustrate their subsequent behavioral investigation, focuses solely on how the donor-priority rule affects an individual's probability of receiving an organ. Our paper, by contrast, builds a queueing model of the organ supply-and-demand processes and makes different assumptions regarding an individual's utility. This different setup allows us to generate rich and interesting insights into the social-welfare consequences of the donorpriority rule. Under Kessler and Roth's (2012) original framework, social welfare increases after the introduction of the donor-priority rule, which is not always the case in our setup.

As a follow-up to their 2012 paper, Kessler and Roth (2014) show through laboratory experiments that, in the presence of a loophole by which an individual may enjoy the donor priority without incurring the cost of donating, the positive incentive effect, as characterized in their earlier work, would disappear. Although our paper does not explicitly model the loophole Kessler and Roth (2014) address, it shares the spirit of revealing unintended incentive distortions resulting from the priority status associated with registered organ donors. Using Israel's organdonor registration data between 1992 and 2013, Stoler et al. (2017) investigate the effect of the donor-priority rule on the pattern of donor registrations. They characterize a significant increase in registration rates approaching the April 1, 2012, deadline, before which registered donors were granted priority on organdonor wait lists without a freeze period. With a perspective different from ours, Stoler et al. (2017) emphasize the role of the effective design of campaigns in raising the awareness of the policy.

Our paper, by building a strategic queueing model of individuals' decisions to become registered organ donors, bridges the operations management and economics literature on organ transplantation. To the best of our knowledge, this paper is the first to jointly incorporate queueing considerations and the cost of donating. Our paper focuses on both the quantity and quality of the pool of donated organs and considers an operational approach to address a complex social problem.

Broadly speaking, our paper is relevant to the rational queueing literature (e.g., Debo et al. 2008, Wang et al. 2010, Anand et al. 2011, Afèche 2013, Kostami and Rajagopalan 2013, Zhan and Ward 2014, Paç and Veeraraghavan 2015, Dai et al. 2017, Guo et al. 2019, Yang and Debo 2019) in that individuals jointly determine the service rate and queue configuration. Without enough donors, the benefit of joining the registry is low because of a limited organ supply; when the number of donors reaches a critical point, the value of priority becomes minimal. Our paper, by being the first to examine the interaction between demand priority and endogenous supply, enriches this literature by characterizing a new trade-off between abundance of supply, exclusivity of priority, and cost of acquiring priority.

3. General Modeling Framework

In this section, we introduce our modeling framework, describing the three key building blocks and laying out the foundation for equilibrium characterization under various organ-donation policies.

At a broad level, each individual may interact with the cadaveric organ-transplantation system by adding to its supply or demand. To reflect this interaction, similar to Kessler and Roth (2012), we say each individual may be at one of three primary states: healthy (in a health status not requiring an organ transplant), sick (in need of an organ transplant), or deceased. Until Section 5, we assume each healthy individual has the same probability of becoming sick or suffering from premature death. In other words, we assume all healthy individuals belong to the same risk type in Sections 3 and 4 and extend this assumption from Section 5 onward. The healthy population arrives at a rate of Λ . Each healthy individual may become sick (and in need of an organ transplant) at a rate of θ or die at a rate of σ . For tractability, we assume exponentially distributed transition times. At the time of an individual's death, with probability ϕ , the individual suffers from an "eligible death" and becomes a potential deceased organ donor, who can provide *n* organs. For simplicity of analysis, we assume an organ transplanted to a recipient will not be used for another transplantation following the recipient's death.

We present three key building blocks of our model: (1) organ supply and demand, (2) a transplant candidate's QALE, and (3) a healthy individual's cost of donating. These building blocks are drawn from the operations and economics literature on organ donation and transplantation. First, in modeling organ supply and demand, we use a fluid-approximation method developed by Zenios (1999), which helps us obtain tractable results with social-welfare implications. Next, in modeling a transplant candidate's pretransplantation and posttransplantation quality of life, we use the QALE model proposed by Su and Zenios (2006). Finally, in modeling a healthy individual's cost of donating, we follow the model developed by Kessler and Roth (2012) to represent the hurdle that an individual must overcome before registering to become an organ donor.

3.1. Modeling Organ Supply and Demand

We use a fluid model to approximate the arrival processes of transplant candidates and cadaveric organs. Each individual may (1) become a transplant candidate (at a rate of θ) or (2) suffer from death (at a rate of σ). The stochastic process by which healthy individuals become transplant candidates is, thus, Poisson as well with a rate, denoted by Θ , of $\frac{\theta}{\theta+\sigma} \cdot \Lambda$. Likewise, the arrival process of potential donated organs resulting from the deaths of all individuals is Poisson with a rate of $\Phi = \frac{\sigma\phi n}{\theta + \sigma} \cdot \Lambda$. To state it differently, Θ represents the total demand rate (the arrival rate of transplant candidates), and Φ represents the total supply rate (the arrival rate of cadaveric organs). We assume $\Phi > \Theta$, meaning the supply rate of organs (from all eligible deaths) is adequate to meet the demand rate from newly added transplant candidates, largely in line with the U.S. organ-transplantation practice: according to the Organ Procurement and Transplantation Network's (OPTN) Deceased Donor Potential Study (Klassen et al. 2016), the potential number of deceased organ donors is approximately 37,258; the potential supply rate is sufficient to serve organ-transplant candidates newly added to the wait list: approximately 12,000 for liver, 35,000 for kid e^{2} 4,000 for heart, and 2,500 for lung.

When a healthy individual evolves to a transplant candidate with probability p_k , that individual belongs to a category k = 1, 2, ..., K; a lower category number indicates a higher level of medical urgency.³ For example, for candidates in need of liver transplants, the categories correspond to their model for end-stage liver disease scores (between 6 and 40); for patients in need of heart transplants, the categories are based on their disease status (1A, 1B, and 2). All candidates line up for organ transplants according to their categories; a queue of a higher-ranking category (i.e., a lower category number) has a higher priority to receive organ transplant. A terminal-category (i.e., category 1) candidate faces either a transplant or death without receiving a transplant. A category-k, k = 2, 3, ..., K, candidate, by contrast, faces one of the following three events: (1) becomes a category (k - 1) candidate at a rate of τ_k , (2) dies at a rate of δ_k , or (3) receives an organ transplant. The mortality rate δ_k decreases in *k*; that is, a higher-ranking category corresponds to a lower pretransplant life expectancy. A category-k candidate's expected time of maintaining the same category without receiving an organ transplant is, thus, $1/(\tau_k + \delta_k)$.

To represent each transplant candidate's utility, we need two performance metrics, namely the candidate's (1) pretransplantation life expectancy and (2) probability of receiving a transplant. Here, we briefly discuss the derivation of fluid approximations based on diffusion limits. Denote by λ_k the arrival rate of individuals of category *k* and by μ_k the arrival rate of organs available to category-*k* candidates. In an asymptotic regime with a scaling factor denoted by *m*, category-*k* candidates arrive at a rate of $m \cdot \lambda_k$, whereas organs available for those candidates arrive at a rate of $m \cdot \mu_k$. If $\mu_k \leq \lambda_k$, for an allocation policy independent of the scaling factor *m*—which is the case both before and after the introduction of the donor-priority rule—as *m* approaches infinity, the system utilization approaches one, giving the asymptotic results (Zenios 1999): approximately, a category-*k* transplant candidate's pretransplantation life expectancy while being of category *k* is

$$\left(\frac{\lambda_k - \mu_k}{\lambda_k}\right)^+ \cdot \frac{1}{\tau_k + \delta_k},\tag{1}$$

and the probability of receiving a transplant while being of category k is

$$\min\left\{1,\frac{\mu_k}{\lambda_k}\right\}.$$
 (2)

Each category-*k* candidate becomes of category (k - 1)at a rate of $\lambda_{k,(k-1)} = (\lambda_k - \mu_k)^+ \cdot \tau_k/(\tau_k + \delta_k)$, so the total arrival rate of transplant candidates of category $k \ge 2$ is $\lambda_k = p_k \Theta + \lambda_{(k+1),k}$ and $\lambda_K = p_K \Theta$. On the other hand, a queue of a higher-ranking category has a higher priority to receive an organ transplant, so the arrival rate of organs available to category-*k* candidates is the residual supply to category (k - 1) candidates; that is, $\mu_k = (\mu_{k-1} - \lambda_{k-1})^+$. In the rest of the paper, we focus on the interesting case in which $\mu_1 - \lambda_1 > 0$, such that category-1 candidates will not be the only category of candidates receiving organ transplant. This case is consistent with the observation that organ transplant is not limited to candidates with the highest level of medical urgency. Our main results hold when $\mu_1 - \lambda_1 < 0$.

3.2. Modeling a Transplant Candidate's QALE

Denote by \hat{D}_k and $\hat{\pi}_k$ a transplant candidate's total life expectancy prior to transplantation and the candidate's total probability of receiving an organ transplant of the candidate starting with category *k*. We obtain $\hat{D}_1 = \left(\frac{\lambda_1 - \mu_1}{\lambda_1}\right)^+ \cdot \frac{1}{\tau_1 + \delta_1}$, $\hat{\pi}_1 = \min\{1, \frac{\mu_1}{\lambda_1}\}$, and the following recursive equations based on the candidate's health-status dynamics while on the wait list:

$$\hat{D}_{k} = \underbrace{\left(\frac{\lambda_{k} - \mu_{k}}{\lambda_{k}}\right)^{+} \cdot \frac{1}{\tau_{k} + \delta_{k}}}_{\text{pre-transplantation life}} \\ + \underbrace{\left(1 - \min\left\{1, \frac{\mu_{k}}{\lambda_{k}}\right\}\right) \cdot \frac{\tau_{k}}{\tau_{k} + \delta_{k}}}_{\text{probability of moving to category } k-1} \cdot \hat{D}_{k-1}, \text{ and} \\ \hat{\pi}_{k} = \hat{\pi}_{k-1} + \min\left\{1, \frac{\mu_{k}}{\lambda_{k}}\right\} \cdot (1 - \hat{\pi}_{k-1})$$

for $k = 2, 3, \dots, K$. A transplant candidate can start with any category $k = 1, \dots, K$. For this reason, from a healthy individual's perspective, the life expectancy from the time of becoming a transplant candidate to the time of transplantation or death (whichever comes first) can be expressed as $D = \sum_{k=1}^{K} p_k \hat{D}_k$, and the probability of receiving an organ transplant if one becomes a candidate can be represented by $\pi = \sum_{k=1}^{K} p_k \hat{\pi}_k$. We use QALE to measure the utility of an individual who is in need of an organ transplant. A candidate's QALE is written as

$$u = \alpha D + \beta \pi T, \tag{3}$$

where α is the candidate's quality-of-life score while on the wait list, β is the candidate's quality-of-life score after transplantation ($0 < \alpha < \beta < 1$), and *T* is the candidate's posttransplantation life expectancy. The QALE equation is from Su and Zenios (2006). The individual's posttransplantation life expectancy is assumed to depend on organ quality, which is determined by the donor's risk type.

3.3. Modeling a Healthy Individual's Cost of Donating

In this section, we describe a healthy individual's decision about whether to join the donor registry.⁴ Each individual incurs a burden from registering as an organ donor, which we—following Kessler and Roth (2012)—refer to as the cost of donating and denote by c.

The cost of donating *c* has a support of $(-\infty, \infty)$, a probability density function of $f(\cdot)$, and a cumulative distribution function of $F(\cdot)$.⁵ Such a cost can be either positive or negative. When an individual has a positive cost of donating (i.e., c > 0), the individual must overcome certain burdens to register to become a potential organ donor. For example, some individuals fear physicians might not try their best to save registered organ donors' lives (Teresi 2012). As another example, certain religious beliefs disfavor the practice of organ donation (Bruzzone 2008). When an individual has a negative cost of donating (i.e., c < 0), the individual derives a positive nonmonetary gain—for example, social recognition and self-fulfillment (Prottas 1983)—from registering to be an organ donor.

Our main findings hold qualitatively under a wide range of distributions of the cost of donating. Although measuring the exact cost of donating is difficult, numerous studies have explored various factors behind a positive or negative cost of donating. We refer readers to Feeley (2007) for a comprehensive survey.

Having presented the three building blocks of our model, we now discuss a healthy individual's decision about whether to join the donor registry. The individual interacts with the organ-transplantation system through becoming either a transplant candidate or a source of donated organs. The individual's decision is reached, therefore, by weighing (1) the supply of and demand for organs, (2) a transplant candidate's QALE, and (3) the cost of donating. Each consideration corresponds to one of our three building blocks. In essence, the trade-off behind the decision to join the donor registry is between the cost of donating and the expected benefit of joining the donor registry. For ease of exposition, in the main body of this paper, we consider the case with two categories of transplant candidates (i.e., K = 2); our main findings carry over to the case with $K \ge 3$. Ceteris paribus, an individual with a higher cost of donating will have a lower incentive to join the donor registry. Thus, given any organ-donation policy, a threshold $C \in (-\infty, \infty)$ exists such that all individuals with $c \leq C$ will register to become organ donors, and all the individuals of risk type *i* with c > C will not. Given a threshold C, the corresponding donation rate (i.e., the proportion of the population who are on the organdonor registry) is F(C). We illustrate our modeling framework in Figure 1.

4. Benchmark: Homogeneous Risk Type

In this section, we study a benchmark in which individuals are heterogeneous in their costs of donating but homogeneous in their risk types. We first derive the social optimum. Then we characterize the equilibria for the cases before and after the introduction of the donor-priority rule, respectively. We compare social welfare across both cases and show that introducing the donor-priority rule leads to improved social welfare.

A threshold cost of donating *C* corresponds to a donation rate of *F*(*C*), which increases in *C*; in other words, a larger *C* corresponds to a higher share of organ donors. We define a constant $\hat{C} \triangleq F^{-1}(\Theta/\Phi)$ such that $\Phi \cdot F(\hat{C}) = \Theta$. In other words, *F*(\hat{C}) is the share of organ donors at which the supply rate of and the demand rate for donated organs are equal to each other.⁶

4.1. Social Optimum

We start with deriving the social optimum. Because all individuals possess the same ex ante expected utility from receiving organ transplants, their contribution to social welfare—irrespective of the allocation scheme—depends solely on their costs of donating. In the social optimum, the donors must be the ones whose costs of donating are sufficiently low. In other words, driving the social optimum entails specifying a welfare-maximizing threshold, denoted by C^{SO}, such that all individuals are on the organ-donor



Figure 1. Illustration of the General Modeling Framework with K = 2 Categories of Transplant Candidates

Note. In the figure, "D" denotes a queue consisting of candidates whose names are on the donor registry, whereas "ND" denotes a queue consisting of candidates whose names are not on the donor registry.

registry if and only if their costs of donating are below C^{SO} .

We define social welfare as the aggregate utility of all individuals. To be more exact, social welfare W_s as a function of *C* can be expressed as the aggregated QALE of all the listed individuals as defined in (3), less the aggregated costs of donating incurred by all the registered organ donors (i.e., the individuals whose cost of donating is lower than *C*). On one hand, if $C \ge \hat{C}$, the arrival rate of organ supply is higher than that of transplant candidates. Building on (3), we have

$$W_{s}(C) = \frac{1}{\theta + \sigma} \cdot \Lambda + \Theta \beta T - \Lambda \int_{-\infty}^{C} cf(c) dc.$$

On the other hand, if $0 \le C < \hat{C}$, the arrival rate of organ supply ($\phi F(C)$) cannot meet the demand rate from all transplant candidates (Θ). In this case, as category-1 candidates are prioritized over category-2 candidates, the arrival rates of transplant candidates (λ_1 and λ_2) and the supply rates of organs available to category-1 and category-2 candidates (μ_1 and μ_2) satisfy the following equations:

$$\lambda_{1} = \Theta p_{1} + \underbrace{(\lambda_{2} - \mu_{2}) \frac{\tau_{2}}{\tau_{2} + \delta_{2}}}_{\text{Evolved from category 2}}, \lambda_{2} = \Theta p_{2};$$
$$\mu_{1} = \Phi F(C), \mu_{2} = \mu_{1} - \lambda_{1}.$$

Jointly solving these equations for λ_1 and μ_2 allows us to represent social welfare as

$$W_{s}(C) = \frac{1}{\theta + \sigma} \cdot \Lambda + \Theta \cdot \frac{\alpha}{\delta_{2}} + \Phi F(C) \left(\beta T - \frac{\alpha}{\delta_{2}}\right)$$

$$-\Lambda \int_{-\infty}^{C} cf(c) dc,$$
(4)

where $1/\delta_2$ in the second and third terms is the life expectancy of a candidate who remains of category 2 (i.e., without receiving organ transplantation or evolving to category 1). In (4), the term $\frac{1}{\theta+\sigma} \cdot \Lambda$ represents the total expected utility of healthy individuals, the term $\Theta \alpha / \delta_2$ is the total expected pretransplantation utility of individuals who become sick, the term $\Phi F(C)(\beta T - \alpha / \delta_2)$ represents social-welfare improvement resulting from organ transplantation, and the term $\Lambda \int_{-\infty}^{C} cf(c)dc$ represents the total costs of donating borne by all registered organ donors. Hence, we have

$$W_{s}(C) = \frac{\Lambda}{\theta + \sigma} - \Lambda \int_{-\infty}^{C} cf(c)dc + \begin{cases} \frac{\theta}{\theta + \sigma}\Lambda \frac{\alpha}{\delta_{2}} + \frac{\sigma\phi n}{\theta + \sigma}\Lambda F(C) \left(\beta T - \frac{\alpha}{\delta_{2}}\right) & \text{if } C < \hat{C}, \\ \frac{\sigma\phi n}{\theta + \sigma}\Lambda\beta T & \text{otherwise.} \end{cases}$$
(5)

Maximizing social welfare gives the socially efficient cost threshold.

Lemma 1. The socially efficient threshold is $C^{SO} = \min \left\{ \frac{\sigma \phi n}{\theta + \sigma} (\beta T - \alpha / \delta_2), \hat{C} \right\}.$

The key takeaway from this lemma is that the socially efficient threshold C^{SO} increases in $(\beta T - \alpha/\delta_2)$: the first term $\frac{\sigma\phi n}{\theta+\sigma}(\beta T - \alpha/\delta_2)$ represents a healthy individual's marginal contribution to the organtransplantation system by registering to become a potential organ donor in the case in which there is a shortage of organ supply; that is, $C < \hat{C}$. Intuitively, the socially efficient threshold is bounded by this marginal contribution. On the other hand, the socially efficient threshold cannot be above C; otherwise, any donor whose cost of donating is above C would contribute to an increase in the total cost of donating without leading to more transplants. Whether *C* is equal to the socially efficient threshold depends on the magnitude of C: under a large C (i.e., the marginal donor bears a high cost of donating to fulfill all demand), the socially efficient cost is lower than \hat{C} . In this case, the socially efficient threshold increases in $(\beta T - \alpha/\delta_2)$.

4.2. Before Introduction of Donor-Priority Rule

Before the introduction of the donor-priority rule, because each individual gains no benefit from registering to become a donor, only those with negative costs of donating (i.e., c < 0) have the incentive to join the organ-donor registry. Thus, in equilibrium, the threshold cost, denoted by C_{np}^* , is zero. Using (5), we represent social welfare in this case as $W_{np} = W_s(0)$.

In the social optimum characterized in Section 4.1, we have $C^{SO} = \min \left\{ \frac{\sigma \phi n}{\theta + \sigma} (\beta T - \alpha / \delta_2), \hat{C} \right\}$. The condition $\beta T > \alpha / \delta_2$ implies that $C^{SO} > 0$. In other words, before the introduction of the donor-priority rule, the donation rate is strictly below the socially efficient level. From the social planner's perspective, some individuals contribute to the organ-transplantation system more than their costs of donating should they register to become organ donors. Nonetheless, without being incentivized, those individuals would not join the donor registry because they are unable to internalize the positive externality.

4.3. After Introduction of Donor-Priority Rule

Under the donor-priority rule, whereas category-1 candidates continue to have priority over category-2 candidates in access to cadaveric organs, within each category, registered donors have priority over nondonors. Thus, two queues arise within each category k = 1, 2: (1) a queue of priority candidates whose names appear on the donor registry and (2) a queue of nonpriority candidates whose names do not

appear on the donor registry. In characterizing the equilibrium, two competing effects exist: First, when an individual decides to register, the individual is, in essence, acquiring an option of joining priority queues should the individual need an organ transplant in the future. Thus, the individual would benefit from a larger organ pool. Second, the marginal value of becoming a donor is diminishing as the donor pool expands because even a nondonor would be able to benefit from an increased organ supply. Together with the cost of donating, each individual's decision to register is ultimately driven by a three-way tradeoff between abundance of supply, exclusivity of priority, and cost of donating.

To characterize the equilibrium, we first represent an individual's utility from registering to become an organ donor. We use C_{v}^{*} to denote the threshold cost of donating at which each individual is indifferent between registering to become a donor and not registering. The supply rate of organs is now $\Phi F(C_v^*)$; the demand rate for organs remains Θ , of which $\lambda_d =$ $F(C_{v}^{*})\Theta$ is the arrival rate of priority candidates (of which $\lambda_{kd} = p_k \cdot \lambda_d$ is the arrival rate of category-*k* priority queue, k = 1, 2), and the remaining $\lambda_n = [1 - 1]$ $F(C_{v}^{*})]\Theta$ is the arrival rate of nonpriority candidates (of which $\lambda_{kn} = p_k \cdot \lambda_n$ is the arrival rate of the category-*k* nonpriority queue, k = 1, 2). Within each category, a priority and a nonpriority queue exist; across categories, category-1 queues have a higher priority than category-2 queues. Similar to the analysis before the introduction of the donor-priority rule, we derive the residual supply rate of organs for category-2 candidates as $\mu_2 = \Theta p_2 - [\Theta - \Phi F(C_v^*)](\tau_2 + \delta_2)/\delta_2$. Because $\mu_2 < \lambda_2 = p_2 \Theta$, category-2 transplant candidates face an organ-supply shortage. Because category-2 priority candidates have a higher priority than category-2 nonpriority candidates, they have different utility values. For conciseness of analysis, in the rest of the paper, we focus on the interesting case in which, beyond category 1, at least one category exists in which some nondonors have access to organs; that is, $\mu_2 > \lambda_{2d}$. Using (1)–(3), we have the following results: (i) an individual's utility from registering to become an organ donor is the individual's QALE from a potential organ transplant, less the cost of donating, that is, $U_d(c) = \frac{1}{\theta + \sigma} + \frac{\theta}{\theta + \sigma} \beta T - c$; and (ii) an individual's utility from not registering can be represented as $U_n(c) = \frac{1}{\theta + \sigma} + \frac{\theta}{\theta + \sigma} \cdot \left\{ \beta T - \frac{\Theta - \Phi F(C_p^*)}{\Theta [1 - F(C_p^*)]} \cdot \left(\beta T - \alpha / \delta_2 \right) \right\}, \text{ which}$ can be derived by noting that an unregistered individual, in the case of becoming a category-2 nonpriority candidate, will face a rationed organ supply.

We can obtain a marginal donor's cost of donating by setting $U_d(c) = U_n(c)$. The following proposition characterizes the threshold cost of donating in equilibrium. **Proposition 1.** In the case in which individuals differ only in their costs of donating, under the donor-priority rule, in equilibrium, only those individuals with cost of donating c below the threshold C_p^* will elect to join the organ-donor registry, where C_p^* satisfies

$$C_p^* = \frac{\theta - \sigma \phi n F(C_p^*)}{(\theta + \sigma) \cdot [1 - F(C_p^*)]} \cdot (\beta T - \alpha / \delta_2).$$
(6)

Such an equilibrium exists and is unique.

Based on the equilibrium characterized in Proposition 1, we can represent social welfare under the donor-priority rule as the aggregate utility of both donors and nondonors:

$$W_p = \Lambda \left[\underbrace{\int_{-\infty}^{C_p^*} U_d(c) f(c) dc}_{\text{Donors'welfare}} + \underbrace{\int_{C_p^*}^{\infty} U_n f(c) dc}_{\text{Non-donors'welfare}} \right].$$

Proposition 1 gives the following corollary:

Corollary 1. $C_p^* > C_{np}^* = 0.$

Corollary 1 implies the donor-priority rule helps expand the size of the donor registry. Prior to the introduction of the donor-priority rule, individuals with positive costs of donating would not register to become donors. Under the donor-priority rule, however, some individuals with positive costs of donating are incentivized to become donors because of the endowed priority of receiving organs should they need organ transplants in the future.

Corollary 2. In the case in which individuals differ only in their costs of donating, under the donor-priority rule, in equilibrium, the threshold cost of donating (C_p^*) increases in the arrival rate of transplant candidates (θ) and decreases in the rate of premature deaths (σ) and the probability that each premature death is eligible for transplantation (ϕ).

This corollary suggests an individual's willingness to join the organ-donor registry increases in the individual's likelihood of requiring an organ transplant. In addition, all else being equal, an individual is less likely to become an organ donor when eligible deaths occur more frequently, providing a more abundant supply of organs.

4.3.1. Comparison with Social Optimum. Corollary 1 implies that introducing the donor-priority rule will lead to a higher donation rate. Nevertheless, we can show the equilibrium donation rate still trails the socially optimal donation rate.

Corollary 3. In the case in which individuals differ only in their costs of donating, under the donor-priority rule, the threshold cost of donating in equilibrium is below that in the social optimum; that is, $C_p^* < C^{SO}$.

Intuitively, an individual, by registering to become a donor, enriches others by increasing the potential organ supply. The donor-priority rule only *partially* internalizes the marginal benefit to social welfare. In other words, a marginal donor's cost of donating (i.e., C_p^*) is equal to the individual's benefit from the donor-priority rule, which is below the marginal benefit to social welfare. Hence, the registration rate in the market equilibrium is below that in the social optimum.

4.4. Social-Welfare Implications

Consider the following three "before-and-after" scenarios: some choose to register to become organ donors regardless of the donation policy; others choose not to register to become organ donors regardless of the donation policy; still others register to become organ donors only under the donor-priority rule. In the first scenario, individuals are better off under the donor-priority rule because they can acquire priority status in accessing an expanded pool of donated organs. Next, individuals in the second scenario are worse off because they rank lower in priority for organ transplants under the donorpriority rule. Finally, individuals in the third scenario will have higher utility than those in the second scenario-otherwise, they would not register to become organ donors-but they may still be better or worse off, depending on their costs of donating. The overall welfare implication of the donor-priority rule is not immediately clear. Our analytical framework helps elucidate the welfare implication and leads to the following proposition.

Proposition 2. Under heterogeneity in costs of donating but not in health status, the introduction of the donor-priority rule always increases social welfare.

Proposition 2 states that introducing the donorpriority rule boosts social welfare. This improvement is principally achieved through an expanded donor pool $(C_v^* > C_{nv}^*)$; the increased supply of donated organs leads to improved QALEs of the overall population. Note that granting donors priority in receiving organs has two effects: an increase in the total costs of donating because a proportion of donors with positive costs are incentivized to register to become donors and an increase in the supply rate of donors and opportunities for organ transplants. Proposition 2 suggests the second effect outweighs the first. The intuition is that if an individual with a positive cost of donating chooses to switch to donating because of the donor-priority rule, the benefit from joining the registry must outweigh the cost of donating. Furthermore, because all individuals are homogeneous in their risk types, the increased organ supply gives rise to a positive externality and, thus, boosts social welfare.

5. Heterogeneity in Both Health Risks and Costs of Donating

In the benchmark presented in the preceding section, we show that, when individuals differ only in terms of costs of donating, introducing the donor-priority rule increases social welfare. In this section, we broaden the scope of our analysis by incorporating a second dimension of heterogeneity: individuals' likelihoods of requiring organ transplants (i.e., risk types).

Specifically, each healthy individual's risk type *i* can be either high (denoted by H) or low (denoted by *L*), which determines the individual's probability of becoming sick or suffering from premature death. The healthy population of risk type $i \in \{L, H\}$ arrives at a rate of Λ_i .⁷ Each type-*i* healthy individual may become sick (and in need of an organ transplant) at a rate of θ_i or die at a rate of σ_i . Using an argument similar to that in Section 3, we can derive the type-*i* healthy individuals' demand rate for organ transplants, denoted by Θ_i , as $\frac{\theta_i}{\theta_i + \sigma_i} \cdot \Lambda_i$ and their supply rate of donated organs, denoted by Φ_i , as $\frac{\sigma_i \phi n^i}{\theta_i + \sigma_i} \cdot \Lambda_i$. We assume a high-risk individual has a higher lifetime likelihood of becoming a transplant candidate than a low-risk individual does; that is, $\frac{\theta_H}{\theta_H + \sigma_H} > \frac{\theta_L}{\theta_L + \sigma_L}$. We use $\Theta = \sum_{i=H,L} \Theta_i$ to represent the total arrival rate of transplant candidates and $\Phi = \sum_{i=H,L} \Phi_i$ to represent the total arrival rate of cadaveric organs.

The risk type of an individual who ends up becoming an organ donor may affect the quality of that individual's donated organs and, thus, the resultant posttransplantation life expectancy. Denote by t_i the length of posttransplantation life after receiving an organ from a type-*i* donor. We assume the life expectancy of a transplant candidate who receives an organ from a type-*L* donor is longer than that from a type-*H* donor; that is, $T_L \triangleq \mathbb{E}[t_L] > T_H \triangleq \mathbb{E}[t_H]$.

In Section 5.1, we characterize the equilibrium before and after the introduction of the donor-priority rule, respectively. In Section 5.2, we derive the social optimum and provide welfare implications of the donor-priority rule. A common feature across all cases is that individuals of different risk types (i = H, L) have different threshold costs of donating, denoted by C_i .

5.1. Equilibrium Characterization

We first consider the case before the donor-priority rule is introduced. As in Section 4.2, only those with negative costs of donating have the incentive to register to become organ donors; in other words, the threshold cost of donating in equilibrium is still $C_{np}^* = 0$ regardless the risk type.

Next, we consider the case in which the donorpriority rule has been introduced. We denote by C_i^* type-*i* individual's threshold cost of donating in equilibrium. We characterize the equilibrium in the following proposition, in which $T_p(C_H^*, C_L^*)$ represents the average posttransplantation QALEs; that is,

$$T_p(C_H^*, C_L^*) = \frac{\sum_{i=H,L} \Phi_i F(C_i^*) T_i}{\sum_{i=H,L} \Phi_i F(C_i^*)}.$$

In the rest of the paper, we use T_p^* and $T_p(C_H^*, C_L^*)$ interchangeably for simplicity of notation.

Proposition 3. In equilibrium, the threshold costs of donating, C_H^* and C_L^* , satisfy

$$\begin{split} C_i^* &= \frac{\Theta_i}{\Theta_i + \sigma_i} \cdot \left[\beta T_p(C_H^*, C_L^*) - \alpha/\delta_2\right] \\ &\cdot \frac{\sum_{j \in \{L, H\}} [\Theta_j - \Phi_j F(C_j^*)]}{\sum_{j \in \{L, H\}} \Theta_j [1 - F(C_j^*)]} \ for \ i = H, L, \end{split}$$

and the preceding equilibrium exists and is unique.

The right-hand side of the condition in Proposition 3 represents type-*i* individuals' expected benefit of registering as an organ donor. Specifically, the first multiplier is the likelihood of needing an organ transplant; the rest captures—when such needs are realized—the benefit of being in a priority queue rather than a nonpriority queue. The proposition immediately gives the following corollary:

Corollary 4. Under the donor-priority rule, in equilibrium, type-H individuals' threshold cost of donating is higher than type-L individuals'; that is, $\frac{C_{H}^{*}}{C_{L}^{*}} = \frac{\theta_{H}/(\theta_{H}+\sigma_{H})}{\theta_{L}/(\theta_{L}+\sigma_{L})} > 1.$

Under the donor-priority rule, the ex post benefit from registering to become an organ donor is the same regardless of an individual's risk type. For this reason, the individual's marginal benefit from registering to become a donor increases in the likelihood of needing an organ transplant in the future. In equilibrium, a marginal donor's cost of donating is equal to the marginal benefit from registering to become a donor. Hence, the result follows.

We define $T_a \triangleq \frac{\sum_{i=H,L} \Phi_i T_i}{\sum_{i=H,L} \Phi_i}$ as a candidate's posttransplant life expectancy from receiving an averagequality organ. Corollary 4, in turn, gives the following corollary:

Corollary 5. Under the donor-priority rule, in equilibrium, a candidate's posttransplant life expectancy from a donated organ is lower than that from an average-quality organ; that is, $T_p(C_{H}^*, C_L^*) < T_a$.

Corollaries 4 and 5 suggest individuals of different risk types perceive the benefit from registering to become organ donors differently and that high-risk individuals are more likely to register. As a result, the average quality of the donated organs is lower than the average quality of donated organs provided that both types of individuals register with an equal likelihood.

5.2. Social-Welfare Implications

Now we examine the welfare consequences of the donor-priority rule in the presence of heterogeneous risk types. Using the equilibrium characterization, we represent social welfare before the introduction of the donor-priority rule as

$$W_{np} = \sum_{i \in \{L,H\}} \left[\frac{1}{\theta_i + \sigma_i} \cdot \Lambda_i + \Theta_i \alpha / \delta_2 + \Phi_i F(0) (\beta T_i - \alpha / \delta_2) - \Lambda_i \int_{-\infty}^0 cf(c) dc \right].$$
(7)

Similarly, we represent social welfare under the donorpriority rule as

$$W_{p} = \sum_{i \in \{L,H\}} \left[\frac{1}{\theta_{i} + \sigma_{i}} \cdot \Lambda_{i} + \Theta_{i} \alpha / \delta_{2} + \Phi_{i} F(C_{i}^{*}) \right.$$

$$\cdot \left(\beta T_{i} - \alpha / \delta_{2}\right) - \Lambda_{i} \int_{-\infty}^{C_{i}^{*}} cf(c) dc \left].$$
(8)

The interpretations for (7) and (8) are rather similar to that for (4).

We observe that social welfare can be characterized as a function of the thresholds C_L and C_H . Maximizing (8) gives both risk types' socially efficient thresholds.

Lemma 2. A type-i individual's socially efficient thresholdcost of donating is $C_i^{SO} = \frac{\sigma_i \phi n}{\theta_i + \sigma_i} (\beta T_i - \alpha / \delta_2)$ for i = L, $H \ if \ \sum_{i \in \{L,H\}} \Phi_i F \left(\frac{\sigma_i \phi n}{\theta_i + \sigma_i} \left(\beta T_i - \alpha / \delta_2 \right) \right) \leq \sum_{i \in \{L,H\}} \Phi_i F (\hat{C}) \triangleq$ $\sum_{i \in \{L,H\}} \Theta_i$.

As in Lemma 1, $\frac{\sigma_i \phi n}{\theta_i + \sigma_i} (\beta T_i - \alpha / \delta_2)$ represents the marginal benefit to the organ-transplantation system as a type-*i* individual registers to become an organ donor when organs are in short supply. Accordingly, the socially efficient threshold (C_i^{SO}) is bounded by this marginal benefit even when demand for organs outweighs supply.

Using Lemma 2 and Proposition 3, we now compare the thresholds in the equilibrium under the donor-priority rule with that in the social optimum.

Corollary 6.
$$\frac{C_H^*}{C_L^*} > \frac{C_H^{SO}}{C_L^{SO}}$$

Corollary 6 reveals a widening gap in terms of willingness to register across risk types because of the introduction of the donor-priority rule. In equilibrium, the ratio of type-H individuals' threshold to type-*L* individuals' is higher than in the social optimum. In other words, the donor-priority rule attracts a disproportionately large number of type-*H* donors vis-à-vis the social optimum.

Corollary 7.

(i) $C_L^* < C_L^{SO}$. (ii) $C_H^* > C_H^{SO}$ if $\frac{\beta T_H - \alpha/\delta_2}{\beta T_P(C_H^*, C_L^*) - \alpha/\delta_2} < \frac{\Theta_H}{\Phi_H} \cdot \frac{\sum_{i \in [L,H]} [\Theta_i - \Phi_i F(C_i^*)]}{\sum_{i \in [L,H]} \Theta_i [1 - F(C_i^*)]}$, which holds when T_H/T_L is sufficiently small.

Corollary 7 suggests a type-L marginal donor's cost of donating is always below the socially efficient level (i.e., the individual's marginal contribution to social welfare). The intuition behind this result is that a type-L individual does not receive a proportional increase in the probability of receiving an organ transplantation from another type-L individual in the case of getting sick and becoming a transplant candidate. On the other hand, we show the type-*H* marginal donor's cost of donating is higher than the socially efficient level when T_H/T_L is sufficiently small. In other words, for high-risk individuals, the donor-priority rule can induce an excessively high threshold cost of donating vis-à-vis the social optimum. The reason is that, by registering to become an organ donor, a type-Hindividual—who is more likely than a type-L individual to need an organ transplant—can enjoy the priority to receive organs from both types of donors. Therefore, under the donor-priority rule, a type-Hindividual is expected to gain more than what the individual contributes to the organ-transplantation system.

Next, we show the introduction of the donor-priority rule can lead to lower social welfare. For ease of exposition, we define

$$\Omega_i(C_i^*) = \Lambda_i \left\{ \int_0^{C_i^*} \frac{\sigma_i \phi n[F(C_i^*) - F(0)]}{\theta_i + \sigma_i} (\beta T_i - \alpha/\delta_2) - \int_0^{C_i^*} cf(c) dc \right\}$$

for $i \in \{L, H\}$ as the change in type-*i* individuals' total contribution to the social welfare resulting from the introduction of the donor-priority rule. Specifically, the first term captures the benefits of increased organ supply, and the second term captures the additional costs of donating.

Proposition 4. Introducing the donor-priority rule leads to a reduction in social welfare if and only if $\Omega_L(C_L^*) < -\Omega_H(C_H^*)$, which holds when $\frac{\theta_L/(\theta_L+\sigma_L)}{\theta_H/(\theta_H+\sigma_H)}$ and T_H/T_L are small enough.

Proposition 4 shows an unintended consequence of the donor-priority rule, which, albeit conducive to expanding the organ-donor registry, may lead to lower social welfare when the additional cost of new donors and the welfare loss of nondonors outweighs the benefit provided by donors. Specifically, $\Omega_L(C_I^*)$ can be interpreted as the net benefit from the increased donation from type-L individuals under the donor-priority rule. This net benefit is always positive as $C_L^* < C_L^{SO}$ as shown in Corollary 6. On the other hand, $-\Omega_H(C_H^*)$ captures the net cost from type-*H* individuals' "over-joining" under the donor-priority rule, which can be positive when $C_H^* > C_H^{SO}$.

Proposition 4 also provides a sufficient condition under which introducing the donor-priority rule reduces the social welfare. To understand the intuition behind this result, let us consider a special case in which $\frac{\theta_L/(\theta_L+\sigma_L)}{\theta_H/(\theta_H+\sigma_H)}$ and T_H/T_L are so small that both $\theta_L/(\theta_L + \sigma_L)$ and $(\beta T_H - \alpha/\delta_2)$ approach zero while $\theta_H/(\theta_H + \sigma_H)$ and $(\beta T_L - \alpha/\delta_2)$ are positive. In this case, according to Proposition 3, C_L^* approaches zero because the donor-priority rule provides little incentive to type-L individuals to register to become organ donors. For this reason, $\Omega_L(C_L^*)$ also approaches zero. On the other hand, C_H^* is positive because the donor-priority rule incentivizes type-H individuals (with positive donating cost) to sign up as organ donors when both $\theta_H/(\theta_H + \sigma_H)$ and T_L are sufficiently large. Hence, the donor-priority rule leads to an increase in the total costs of donating of type-Hindividuals. Nonetheless, as $(\beta T_H - \alpha/\delta_2)$ approaches zero, the benefit from an increased number of type-Hdonors is negligible, which suggests an overall positive net cost from incentivizing type-*H* individuals; that is, $-\Omega_H(C_H^*) > 0$. Thus, $\Omega_L(C_L^*) < -\Omega_H(C_H^*)$ holds. Following a similar intuition, by continuity, we can show the preceding condition holds when $\frac{\theta_L/(\theta_L+\sigma_L)}{\theta_H/(\theta_H+\sigma_H)}$ and T_H/T_L are small enough.

Throughout this paper, we have assumed that organ quality differs across risk types; that is, $T_H < T_L$. In Section OA.5 of the online appendix, we consider a case in which $T_H = T_L$ and show, in that case, social welfare can nevertheless decrease under the donorpriority rule.

6. Freeze-Period Remedy

We have shown the introduction of the donor-priority rule can lead to a reduction in social welfare because of an imbalanced incentive structure formed among individuals of heterogeneous risk types. In this section, we consider a simple and easy-to-implement freeze-period remedy. Under the remedy, registered organ donors do not enjoy priority in the access to organ transplants until they have been on the registry for a specified period of time, which we refer to as a freeze period. We show that the remedy can effectively mitigate the quality-distorting effect as a result of the donorpriority rule. We also prove that this remedy, when used in conjunction with the donor-priority rule, can ensure social-welfare improvement by expanding the size of the donor registry without reducing the average quality of donated organs or inducing unnecessarily high costs of donating.

We denote by *S* the freeze period. Because of the Poisson property of the stochastic processes (see Section 3 for details), the time an individual remains healthy satisfies an exponential distribution with a mean of $1/(\theta_i + \sigma_i)$. Thus, the individual remains healthy with a probability of $e^{-(\theta_i + \sigma_i)S}$ during the freeze period. Under the freeze-period remedy, we can show that each individual uses a threshold policy in determining whether to sign up for organ-donor registry such that individuals of type *i* choose to register to become organ donors if and only if their cost of donating is below a threshold denoted by C_i^{\sharp} . The aggregated posttransplantation life expectancy for the patients receiving organs is

$$T_P^{\sharp} \triangleq T_p(C_H^{\sharp}, C_L^{\sharp}) = \frac{\sum_{i=H,L} \Phi_i F(C_i^{\sharp}) T_i}{\sum_{i=H,L} \Phi_i F(C_i^{\sharp})}$$

Denote by C_i^{\sharp} type-*i* individuals' threshold costs of donating for *i* = *H*, *L*. The following proposition characterizes the equilibrium.

Proposition 5. In equilibrium, the threshold costs of donating C_i^* , i = H, L satisfy

$$C_{i}^{\sharp} = \frac{\theta_{i}e^{-(\theta_{i}+\sigma_{i})S}}{\theta_{i}+\sigma_{i}} \cdot \left[\beta T_{p}(C_{H}^{\sharp}, C_{L}^{\sharp}) - \alpha/\delta_{2}\right]$$
$$\cdot \frac{\sum_{j \in \{L,H\}} \left[\Theta_{j} - \Phi_{j}F(C_{j}^{\sharp})\right]}{\sum_{i \in \{L,H\}} \Theta_{i}\left[1 - F(C_{i}^{\sharp})e^{-(\theta_{i}+\sigma_{j})S}\right]} > 0.$$

Comparing Proposition 5 with Proposition 3, we observe that imposing a freeze period reduces the likelihood of enjoying the priority associated with the donor status and increases the arrival rates of the nonpriority queues.

The following corollary immediately follows from Proposition 5:

Corollary 8. Under the donor-priority rule complemented by a freeze period *S*, in equilibrium, the threshold cost of donating C_i^{\sharp} , i = H, L decreases in *S* and satisfies $\frac{C_H^{\sharp}}{C_L^{\sharp}} = \frac{\theta_H/(\theta_H + \sigma_H)}{\theta_L/(\theta_L + \sigma_L)} \cdot e^{-(\theta_H + \sigma_H - \theta_L - \sigma_L)S}$.

Next, we characterize the freeze period under which the average quality of donated organs is the same as the population average. We refer to such a freeze period, denoted by S^{QR} , as the "qualityrestoring freeze period."

Corollary 9. Under a freeze period of $S^{QR} = \frac{\ln\left(\frac{\theta_H}{\theta_H + o_H}\right) - \ln\left(\frac{\theta_L}{\theta_L + o_L}\right)}{\theta_H + \sigma_H - \theta_L - \sigma_L}$, the average quality of the pool of donated organs in the equilibrium is the same as the population average; that is, $\frac{C_H^2}{C_L^4} = 1$ and $T_p(C_H^{\sharp}, C_L^{\sharp}) = T_a = \frac{\sum_{i=H,L} \Phi_i T_i}{\sum_{i=H,L} \Phi_i}$.

Corollaries 8 and 9 imply the existence of a unique quality-restoring freeze period, which

eliminates distorted incentives resulting from the donor-priority rule. However, whether imposing such a quality-restoring freeze period along with the donor-priority rule would indeed improve social welfare remains unclear because social welfare depends on both the benefits from organ transplants and the costs of donating.

The following proposition provides implications as to whether the quality-restoring freeze period improves social welfare.

Proposition 6. The quality-restoring freeze period, when enforced along with the donor-priority rule, leads to better social welfare than before the introduction of the donorpriority rule.

Proposition 6 shows the donor-priority rule, when implemented alongside the quality-restoring freeze period, always leads to increased social welfare. The intuition is that the quality-restoring freeze period counteracts the asymmetric incentive structure induced by the donor-priority rule. Hence, it increases the total organ supply without compromising the average quality of the organ supply. Furthermore, it discourages individuals with both high risks and high costs of donating from inefficiently registering to become organ donors.

Proposition 7. *A finite optimal freeze period exists, under which introducing the donor-priority rule always leads to an increase in social welfare.*

Proposition 7 shows the existence of a finite optimal freeze period and, more importantly, that such a freeze period can guarantee an improvement in social welfare if the freeze-period remedy is implemented in conjunction with the donor-priority rule.

We close this section by pointing out that the freezeperiod remedy may be viewed as a friction to the social system. The imbalanced incentive structure resulting from the heterogeneous health status puts the donor-priority rule in a second-best situation. The freeze-period remedy is another distortion that disincentivizes all individuals to register to become organ donors. Interestingly, introducing it counteracts the asymmetric incentives and results in higher social welfare.

7. Numerical Illustration

In this section, we present a numerical analysis to illustrate and complement our results from Sections 5 and 6.

As a first step, we estimate those parameters in our model roughly based on the U.S. liver-transplantation system. Each year, there are approximately four million births in the United States, so we assume the arrival rate of individuals to be $\Lambda = 4$ million/year. We use $\Lambda_H =$

 $0.2\Lambda = 0.8$ million/year and $\Lambda_L = 0.8\Lambda = 3.2$ million/ year for illustration purposes. To estimate the arrival rate of candidates requiring a lifesaving organ transplant, we refer to the national transplant data from OPTN (http://optn.transplant.hrsa.gov). We approximate the arrival rate of transplant candidates as Θ = 12,000 per year, close to 11,415 new candidates per year from the OPTN national liver-transplantation data (2011–2015). To fit the arrival rates of both healthy individuals and transplant candidates, we choose θ_i and σ_i , $i \in \{H, L\}$, such that $\theta_H/(\theta_H + \sigma_H) = 0.0098$ and $\theta_L/(\theta_L + \sigma_L) = 0.0013$. Based on the OPTN Deceased Donor Potential Study (https://optn.transplant .hrsa.gov/media/1161/ddps_03-2015.pdf), there are around 38,000 potential donors per year (i.e., around 0.95% of four million), so we approximate the probability of premature brain death by $\phi = 0.0095$. We choose n = 0.392 to fit the data on the number of liver transplants from OPTN; our numerical result suggests that, before the introduction of the donorpriority rule, 6,267 transplants are performed per year, which is close to 6,273 transplants per year from the OPTN national liver-transplantation data (2011 - 2015).

We follow Ouwens et al. (2003) by assuming the quality of life after transplantation is $\beta = 0.75$, and that of the patients on the wait list is $\alpha = 0.5$. We use the data from Said et al. (2004) to estimate the underlying exponential distribution, and the result suggests the pretransplant life expectancy is $1/\delta_2 = 5.83$. We use the OPTN liver survival-rate data for all the donor types and the best exponential fit to estimate posttransplant life expectancy at 16.5 years. Accordingly, we assign the posttransplant expected survival after receiving an organ from a high-risk individual to be $T_H = 10$ and that from a low-risk individual to be $T_L = 18$. To quantify the social-welfare consequences, we assume the economic value per quality-adjusted lifeyear to be \$50,000 following Diamond and Kaul (2009). We assume the cost of donating satisfies normal distribution with a mean of \$600 or economic value of 0.012 quality-adjusted life-year and a standard deviation of \$2,971.86 or economic value of around 0.0594 qualityadjusted life-year, which results in a donor sign-up rate of around 42% before the introduction of the donor-priority rule. In addition, our main findings hold qualitatively under a wide range of distributions of the cost of donating.

Figure 2 illustrates how introducing the freeze period affects social welfare: as the duration of the freeze period increases, social welfare first increases and then decreases. The reason is that when the freeze period is short, the benefit from reducing the asymmetric incentives dominates the cost of reducing organ supply; the opposite is true when the freeze **Figure 2.** (Color online) Impact of the Duration of the Freeze Period on Social Welfare



period is sufficiently long. Interestingly, we observe that a relatively brief freeze period (e.g., one to three years) can result in a steep increase in social welfare, which is sufficient to overcome the reduction in social welfare induced under the donor-priority rule. A threeyear freezing period (as in the case of Israel), for example, boosts the change in social welfare from a reduction of \$75.91 million/year to an increase of \$234.89 million/year. The welfare-maximizing freezing period is 7.27 years, which results in an increase in social welfare of \$279.35 million/year.

In addition, we explore the potential in socialwelfare improvement by examining the ratio of social welfare under the optimal freeze period to that under the social optimum. Table 1 shows significant room exists for social-welfare improvement when the mean cost of donating is high. One implication from Table 1 is that policies such as the donor-priority rule should be complemented with initiatives to help individuals overcome their cultural, psychological, and social barriers to organ donation. Efforts to enhance public awareness (e.g., Facebook's sharing function or nudging as described in Section 1) can play a complementary role in this area.

8. Extensions

In this section, we consider several extensions to explore the boundary of our model and key findings. In Section 8.1, we consider the case in which transplant candidates may opt to turn down an organ offer. In Section 8.2, we consider the case in which an individual of low-risk type can undertake a risky action to become of high-risk type. In Section 8.3, we consider the intertemporal dynamics in an individual's decision to register to become an organ donor. We briefly summarize our findings from these extensions in this section and relegate our analysis to Sections OA.1–OA.3 of the online appendix.

8.1. Candidate Autonomy

In our baseline model, we assume away the possibility of candidate autonomy to focus on broader welfare consequences of the donor-priority rule. We now relax this assumption to allow transplant candidates to decline offered organs. Compared with the case without candidate autonomy, the threshold costs of donating are lower in the presence of candidate autonomy, whereas the ratio between the thresholds of high- and low-risk individuals remains the same. The implication is that the presence of candidate autonomy effectively decreases the total supply of organs but does not change the asymmetric incentives between individuals with heterogeneous health status. Because the donor-priority rule results in asymmetric incentives, introducing the donor-priority rule can reduce social welfare as in the case of the main model.

8.2. Moral Hazard

In the main model, we assume each individual's risk type is exogenous. In this section, drawing from the health economics literature on risky health behavior (e.g., Cawley and Ruhm 2012), we endogenize each individual's risk type by allowing a low-risk individual to undertake a risky action to become of high risk. Our key findings from this extension are twofold: First, introducing the donor-priority rule still motivates more people to sign up as organ donors, and the incentives are stronger for risk-taking individuals. Second, introducing the donor-priority rule can

Table 1. The Ratio of Social Welfare (Relative to the Status Quo) Under the Optimal Freeze

 Period to That Under the Social Optimum

Mean cost of donating, \$	800	750	700	650	600	550	500	450	400	350	300
Ratio of social welfare, %	20.17	22.01	24.20	26.74	29.62	32.77	36.13	39.73	43.68	55.05	72.56
Note. We vary the mean cost of donating from \$800 to \$300 and adjust the corresponding standard											

deviation of the normal distribution to maintain the same donor sign-up rate before the introduction of the donor-priority rule.

induce more people to take risks, especially those individuals who choose to register to become organ donors.

8.3. Dynamics in Decision to Register

In the main model, we assume each individual's risk type does not change over time. Because the registration decision is determined by the risk type and the donation cost, the decision to donate involves no intertemporal dynamics. We relax this assumption and allow each low-risk individual to transition into a high-risk one over time. We characterize the intertemporal dynamics in each individual's decision to register, which allows us to show that certain individuals (i.e., low-risk individuals whose costs of donating are not too low or too high) are inclined to wait and only sign up as organ donors if they become of high-risk type.

9. Concluding Remarks

In this paper, we model and analyze the donor-priority rule, an initiative aimed at expanding the organ-donor registry. Under the donor-priority rule, registered organ donors enjoy queueing priority over nondonors should they need organ transplants in the future. The inner workings of the donor-priority rule present a compelling venue for queueing theoretic analysis with a three-way trade-off between abundance of supply, exclusivity of priority, and cost of donating. To the best of our knowledge, the queueing literature has not examined this type of problem before.

Our analysis shows society would indeed be better off when individuals differ only in their costs of donating (Section 4). Furthermore, our analysis of the heterogeneous-population case in Section 5 reveals an unbalanced incentive structure induced by the donor-priority rule. As a result, although the initiative induces a more sizable organ-donor registry, the average quality of the donated organs can be lower because of the distorted incentives. We proceed to consider an operational remedy that entails enforcing a freeze period, that is, a specified delay in granting individuals priority on wait lists for organ transplants. We show the freeze period provides a disincentive for both types of individuals to become organ donors, yet the disincentive is stronger for high-risk individuals than for low-risk individuals. Thus, appropriately choosing the length of the freeze period can mitigate the quality-distorting effect introduced by the donor-priority rule. We show that this second market distortion, in conjunction with the donorpriority rule, can ensure an increase in social welfare by boosting the supply of organs without sacrificing the quality.

Our model has several limitations. In our model, each individual's posttransplantation life expectancy

is assumed to depend on organ quality determined by the donor's risk type. If we allow the posttransplantation life expectancy to depend on both the donor's risk type and the candidate's category, we can show that individuals will have a stronger incentive to become organ donors under the donorpriority rule. The reason is that candidates desire transplantation while they are relatively healthy, so the value of priority is higher. Nonetheless, as long as the difference in posttransplantation life expectancy across categories is not exceedingly large, by continuity, introducing the donor-priority rule can still reduce social welfare. Another limitation of our model is that we do not explicitly consider that organs can be discarded as a result of uncontrollable circumstances. This consideration can be incorporated into the model by adding another parameter representing the probability that an organ is discarded, which may depend on the donor's risk type. The difference in the probability that an organ is discarded among different types quantitatively affects our results in a similar way as the difference in the probability of brain death or the average number of organs per deceased donor among different types.

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Appendix. Proofs

Proof of Lemma 1. When $C < \hat{C}$, we have from (5) that $dW_s(C)/dx = [\Phi(\beta T - \alpha/\delta_2) - \Lambda C]f(C) = \Lambda[\frac{\sigma\phi n}{\theta+\sigma}(\beta T - \alpha/\delta_2) - C]f(C)$. Thus, the regional maximum for $C \in (-\infty, \hat{C})$ is attained at $\frac{\sigma\phi n}{\theta+\sigma}(\beta T - \alpha/\delta_2)$ or \hat{C} , whichever is lower. If $C > \hat{C}$, we observe from (5) that $W_s(C)$ decreases in C, indicating the regional maximum for $C \in (\hat{C}, \infty)$ is achieved at \hat{C} . Taken together, the socially optimal threshold C^{SO} is $\frac{\sigma\phi n}{\theta+\sigma}(\beta T - \alpha/\delta_2)$ or \hat{C} , whichever is lower. Q.E.D.

Proof of Proposition 1. An individual with the threshold cost C_p^* is indifferent between joining the donor registry or not; that is, $\frac{\theta}{\theta+\sigma}\beta T - C_p^* = \frac{\theta}{\theta+\sigma}\left\{\beta T - \frac{\Theta-\Phi F(C_p^*)}{\Theta[1-F(C_p^*)]}\left(\beta T - \alpha/\delta_2\right)\right\}$. This equation, after rearrangement of its terms, gives (6). The left-hand side of Equation (6) is increasing in C_p^* , whereas its right-hand side can be rewritten as

$$\frac{\theta - \sigma \phi n F(C_p^*)}{(\theta + \sigma)[1 - F(C_p^*)]} \cdot (\beta T - \alpha/\delta_2)
= \left\{ \frac{\sigma \phi n}{\theta + \sigma} + \frac{\theta - \sigma \phi n}{(\theta + \sigma)[1 - F(C_p^*)]} \right\} \cdot (\beta T - \alpha/\delta_2),$$
(A.1)

which is decreasing in C_p^* because $\Phi > \Theta$ gives $\theta - \sigma \phi n < 0$. Furthermore, when $C_p = 0$, LHS = 0 < RHS, and $\lim_{C_p \to \infty} LHS = \infty > -\infty = \lim_{C_p \to \infty} RHS$. Hence, a unique solution $C_p^* > 0$ to (A.1) exists. *Q.E.D.*

Proof of Corollary 2. We have from (6) that

$$\sigma\phi n - \theta = \left(\frac{\theta}{\theta + \sigma} - \frac{C_p^*}{\beta T - \alpha/\delta_2}\right) \left[\frac{1}{F(C_p^*)} - 1\right],\tag{A.2}$$

where the right-hand side is decreasing in C_p^* and σ but increasing in θ , and the left-hand side is decreasing in θ but increasing in σ and ϕ . As θ increases, the left-hand side of (A.2) decreases, requiring a higher C_p^* to balance the equation. Similarly, we can show that C_p^* decreases with σ and ϕ . *Q.E.D.*

Proof of Corollary 3. Recall that $C^{SO} = \min \left\{ \frac{\sigma \phi n}{\theta + \sigma} (\beta T - \alpha/\delta_2), \hat{C} \right\}$. On the one hand, as $\theta - \sigma \phi n F(C_p^*) > 0$ [from (6)], we have $C_p^* < \hat{C} = F^{-1}(\theta/(\phi n))$. On the other hand, we have

$$C_{p}^{*} = \frac{\theta - \sigma \phi n F(C_{p}^{*})}{(\theta + \sigma)[1 - F(C_{p}^{*})]} (\beta T - \alpha/\delta_{2})$$
$$= \left\{ \frac{\sigma \phi n}{\theta + \sigma} + \frac{\theta - \sigma \phi n}{(\theta + \sigma)[1 - F(C_{p}^{*})]} \right\} \cdot (\beta T - \alpha/\delta_{2})$$
$$< \frac{\sigma \phi n}{\theta + \sigma} (\beta T - \alpha/\delta_{2})$$

because $\theta < \sigma \phi n$. Hence, we have $C_p^* < \min \left\{ \frac{\sigma \phi n}{\theta + \sigma} (\beta T - \alpha / \delta_2), \hat{C} \right\} = C^{SO}$. Q.E.D.

Proof of Proposition 2. We examine the difference in social welfare before and after the introduction of the donor-priority rule: $W_p - W_{np} = \Lambda \left[\frac{\theta - F(0)\sigma\phi n}{\theta + \sigma} (\beta T - \alpha/\delta_2) + E(c|c \le 0)F(0) \right] - \Lambda \left\{ E(c|c \le C_p^*)F(C_p^*) + C_p^*[1 - F(C_p^*)] \right\}$, which, by Proposition 1, can be rewritten as

$$W_{p} - W_{np} = \Lambda \bigg\{ C_{p}^{*} [F(C_{p}^{*}) - F(0)] \frac{1 - F(C_{p}^{*})}{\theta / (\sigma \phi n) - F(C_{p}^{*})} - \int_{0}^{C_{p}^{*}} cf(c) dc \bigg\}.$$

Now, because $\theta < \sigma \phi n$, we have $[1 - F(C_p^*)] / [\theta / (\sigma \phi n) - F(C_p^*)] >$ 1, which gives $W_p - W_{np} > C_p^* [F(C_p^*) - F(0)] - \int_0^{C_p^*} cf(c) dc > 0.$ That is, social welfare improves under the donor-priority rule. *Q.E.D.*

Proof of Proposition 3. Consider a type-*i* individual with the threshold C_i^* , i = H, L. We first derive the utility of a priority candidate. The arrival rate of type-*i* priority candidates is $\lambda_d^i = \Theta_i F(C_i^*)$ and that of type-*i* nonpriority candidates is $\lambda_n^i = \Theta_i [1 - F(C_i^*)]$ for i = H, L. In addition, the total arrival rate of organs is $\mu = \sum_{i=H,L} \Phi_i F(C_i^*)$. In this case, the arrival rates of transplant candidates and of organ supply, specified at the category level, are determined endogenously by the following equations:

$$\begin{split} \lambda_1 &= \sum_{i \in \{L,H\}} \Theta_i p_1 + (\lambda_2 - \mu_2) \frac{\tau_2}{\tau_2 + \delta_2}, \lambda_2 = \sum_{i \in \{L,H\}} \Theta_i p_2; \\ \mu_1 &= \sum_{i \in \{L,H\}} \Phi_i F(C_i^*), \mu_2 = \mu_1 - \lambda_1. \end{split}$$

Jointly solving these equations gives

$$\lambda_1 = \sum_{i \in \{L,H\}} \Theta_i p_1 + \frac{\tau_2}{\delta_2} \sum_{i \in \{L,H\}} [\Theta_i - \Phi_i F(C_i^*)] \text{ and}$$
$$\mu_2 = \sum_{i \in \{L,H\}} \Theta_i p_2 - \frac{\tau_2 + \delta_2}{\delta_2} \sum_{i \in \{L,H\}} [\Theta_i - \Phi_i F(C_i^*)].$$

Category-2 priority candidates are granted priority in receiving those available organs over category-2 nonpriority candidates; that is, the total supply rate of organs available to category-2 priority candidates is $\mu_{2d} = \mu_2$, which is greater than the total arrival rate of category-2 priority candidates, $\lambda_{2d} = \sum_{i \in \{L,H\}} \lambda_{2d}^i$. Hence, by registering to become an organ donor, a type-*i* individual with a cost of donating *c* obtains a net utility of

$$U_d^i(c) = \frac{1}{\theta_i + \sigma_i} + \frac{\theta_i}{\theta_i + \sigma_i} \beta T_p(C_H^*, C_L^*) - c.$$

Next, we derive the utility of a nonpriority candidate. The arrival rate of category-2 nonpriority candidates is $\lambda_{2n} = \sum_{i=H,L} \lambda_{2n}^i$, and the total supply rate of organs available to them is $\mu_{2n} = \mu_2 - \lambda_{2d} = \sum_{i \in \{L,H\}} \Theta_i p_2 [1 - F(C_i^*)] - \frac{\tau_2 + \delta_2}{\delta_2} \sum_{i \in \{L,H\}} [\Theta_i - \Phi_i F(C_i^*)]$. By using (1) and (2), we represent each category-2 nonpriority candidate's pretransplantation life expectancy and probability of receiving an organ transplant as

$$D_{2n} = \frac{\sum_{i \in \{L,H\}} [\Theta_i - \Phi F(C_i^*)]}{\sum_{i \in \{L,H\}} \Theta_i p_2 [1 - F(C_i^*)]} \cdot \frac{\tau_2 + \delta_2}{\delta_2} \cdot d_2 \text{ and}$$
$$\pi_{2n} = 1 - \frac{\sum_{i \in \{L,H\}} [\Theta_i - \Phi_i F(C_i^*)]}{\sum_{i \in \{L,H\}} \Theta_i p_2 [1 - F(C_i^*)]} \cdot \frac{\tau_2 + \delta_2}{\delta_2}.$$

Therefore, by not registering to become an organ donor, a type-*i* individual derives a utility of

$$\begin{split} U_n^i = & \frac{1}{\theta_i + \sigma_i} + \frac{\theta_i}{\theta_i + \sigma_i} \bigg\{ \beta T_p^* - \frac{\sum_{j \in \{L,H\}} [\Theta_j - \Phi_j F(C_j^*)]}{\sum_{j \in \{L,H\}} \Theta_j [1 - F(C_j^*)]} \\ & \cdot \Big(\beta T_p^* - \alpha/\delta_2 \Big) \frac{\sum 1}{\theta_i} \bigg\}. \end{split}$$

We can derive the threshold costs of donating (i.e., the cost of donating of a type-*i* marginal donor for i = H, L) by setting $U_d^i(c) = U_n^i$ at $c = C_i^*$, which can be rewritten as

$$\begin{split} C_i^* &= \frac{\theta_i}{\theta_i + \sigma_i} \frac{\sum_{j \in \{L, H\}} [\Theta_j - \Phi_j F(C_j^*)]}{\sum_{j \in \{L, H\}} \Theta_j [1 - F(C_j^*)]} \left[\beta T_p(C_H^*, C_L^*) - \alpha / \delta_2 \right] \\ & \text{for } i = H, L. \end{split}$$

It is straightforward to observe that $\frac{C_{L}^{*}}{\frac{\partial_{L}}{\partial_{L}+\sigma_{L}}} = \frac{C_{H}^{*}}{\frac{\partial_{H}}{\partial_{H}+\sigma_{H}}}$ in equilibrium because, otherwise, if $\frac{C_{L}^{*}}{\frac{\partial_{L}}{\partial_{L}+\sigma_{L}}} \neq \frac{C_{H}^{*}}{\frac{\partial_{H}}{\partial_{H}+\sigma_{H}}}$, the equation suggests

$$\begin{split} & \frac{\sum_{j \in \{L,H\}} [\Theta_j - \Phi_j F(C_j^*)]}{\sum_{j \in \{L,H\}} \Theta_j [1 - F(C_j^*)]} \left[\beta T_p(C_H^*, C_L^*] - \alpha/\delta_2 \right) \\ & \neq \frac{\sum_{j \in \{L,H\}} [\Theta_j - \Phi_j F(C_j^*)]}{\sum_{j \in \{L,H\}} \Theta_j [1 - F(C_j^*)]} \left[\beta T_p(C_H^*, C_L^*) - \alpha/\delta_2 \right], \end{split}$$

resulting in a contradiction. Denoting $y^* = \frac{C_i^*}{\frac{\theta_i}{\theta_i + \sigma_i}}$, the original problem is equivalent to finding a y^* satisfying

$$y = \left[\beta T_p \left(\frac{\theta_H}{\theta_H + \sigma_H} y, \frac{\theta_L}{\theta_L + \sigma_L} y\right) - \alpha/\delta_2\right]$$
$$\cdot \frac{\sum_{j=H,L} \left[\Theta_j - \Phi_j F(\frac{\theta_j}{\theta_j + \sigma_j} y)\right]}{\sum_{j=H,L} \Theta_j \left[1 - F(\frac{\theta_j}{\theta_j + \sigma_j} y)\right]},$$

in which the left-hand side is increasing in *y*, and the righthand side is decreasing in y when y is not too large. Intuitively, the life expectancy posttransplantation $T_p(\frac{\theta_H}{\theta_H + \sigma_H}y, \frac{\theta_L}{\theta_L + \sigma_L}y)$ is decreasing in y as a higher y asymmetrically increases organ supply from the high-risk type. The term $\frac{\sum_{j=H,L} p_j[\Theta_j - F(\frac{\Theta_j}{\partial_j + \sigma_j y})\Phi_j]}{\sum_{j=H,L} \Theta_j[1 - F(\frac{\Theta_j}{\partial_j + \sigma_j y})]}$ measures the probability that nondonors cannot receive organ transplantation, which decreases with y. Moreover, 0 = LHS < RHS when y = 0, and LHS > RHS = 0 when y > 0satisfies $\sum_{j=H,L} [\Theta_j - \Phi_j F(\frac{\theta_j}{\theta_j + \sigma_j} y)] = 0$. Hence, the solution $y^* > 0$ exists and is unique, and so does (C_L^*, C_H^*) . *Q.E.D.*

Proof of Corollary 5. The result follows by using $T_p(C_H^*, C_L^*) =$ $\frac{\sum_{i=H,L} \Phi_i F(C_i^*) T_i}{\sum_{i=H,L} \Phi_i F(C_i^*)}, C_H^* > C_L^*, \text{ and } T_H < T_L. \quad Q.E.D.$

Proof of Proposition 4. We examine the difference in social welfare before and after the introduction of the donor-priority rule:

$$\begin{split} W_p - W_{np} &= \sum_{i=H,L} \Lambda_i \frac{[F(C_i^*) - F(0)]\sigma_i \phi n}{\theta_i + \sigma_i} (\beta T_i - \alpha/\delta_2) \\ &- \sum_{i=H,L} \Lambda_i \int_0^{C_i} cf(c) dc, \end{split}$$

which is negative if and only if $\sum_{i=H,L} \Lambda_i \frac{[F(C_i)-F(0)]\sigma_i\phi_n}{\theta_i+\sigma_i}$. $(\beta T_i - \alpha/\delta_2) < \sum_{i=H,L} \Lambda_i \int_0^{C_i} cf(c)dc$, which can be rewritten as $\Omega_L(C_L^*) < - \Omega_H(C_H^*)$.

From Proposition 3, we can obtain $\lim_{\frac{\theta_L}{\partial_L + \alpha_L} \to 0} C_L^* = 0$, which suggests $\lim_{\substack{\theta_L \\ \theta_L + \alpha_L}} \Omega_L(C_L^*) = 0$. On the other hand, when $\frac{\theta_L}{\theta_L + \alpha_L}$ and $(\beta T_H - \alpha/\delta_2)$ approach zero, from Proposition 3,

$$\begin{split} C_{H}^{*} &= \frac{\theta_{H}}{\theta_{H} + \sigma_{H}} \frac{\sum_{j \in \{L,H\}} \left[\Theta_{j} - \Phi_{j}F(C_{j}^{*})\right]}{\sum_{j \in \{L,H\}} \Theta_{j}[1 - F(C_{j}^{*})]} \left(\beta T_{p}(C_{H}^{*}, C_{L}^{*}) - \alpha/\delta_{2}\right) \\ &= \frac{\theta_{H}}{\theta_{H} + \sigma_{H}} \frac{\sum_{j \in \{L,H\}} \left[\Theta_{j} - \Phi_{j}F(C_{j}^{*})\right]}{\sum_{j \in \{L,H\}} \Theta_{j}[1 - F(C_{j}^{*})]} \\ &\cdot \frac{\Phi_{L}F(C_{L}^{*})(\beta T_{L} - \alpha/\delta_{2}) + \Phi_{H}F(C_{H}^{*})(\beta T_{H} - \alpha/\delta_{2})}{\Phi_{L}F(C_{L}^{*}) + \Phi_{H}F(C_{H}^{*})}, \end{split}$$

approaches $\frac{\theta_H}{\theta_H + \sigma_H} \frac{[\Theta_L - \Phi_L F(0)] + [\Theta_H - \Phi_H F(C_H^*)]}{\Theta_L [1 - F(0)] + \Theta_H [1 - F(C_H^*)]} \frac{\Phi_L F(0) (\beta T_L - \alpha / \delta_2)}{\Phi_L F(0) + \Phi_H F(C_H^*)}$. Let C_{H}^{\lim} represent the solution to

$$\begin{split} C_{H}^{\text{lim}} &= \frac{\theta_{H}}{\theta_{H} + \sigma_{H}} \frac{[\Theta_{L} - \Phi_{L}F(0)] + \left[\Theta_{H} - \Phi_{H}F(C_{H}^{\text{lim}})\right]}{\Theta_{L}[1 - F(0)] + \Theta_{H}[1 - F(C_{H}^{\text{lim}})]} \\ &\cdot \frac{\Phi_{L}F(0)(\beta T_{L} - \alpha/\delta_{2})}{\Phi_{L}F(0) + \Phi_{H}F(C_{H}^{\text{lim}})} > 0, \end{split}$$

so we obtain $\lim_{\frac{\theta_L}{\theta_L + \alpha_L} \to 0} \lim_{\beta T_H - \alpha/\delta_2 \to 0} C_H^* = C_H^{\lim} > 0$. Moreover, $\frac{\sigma_{H}\phi_{H}[F(C_{H})-F(0)]}{\theta_{H}+\sigma_{H}}(\beta T_{H}-\alpha/\delta_{2}) \text{ approaches zero when } (\beta T_{H}-\alpha/\delta_{2})$ $\begin{array}{l} \alpha/\delta_2) \text{ approaches zero. Hence, } \lim_{\substack{\theta_L \\ \sigma_L < \sigma_L \end{pmatrix}} \lim_{\substack{\theta_L < \sigma_L \\ \sigma_L < \sigma_L \end{pmatrix}} \lim_{\substack{\theta_L \\ \sigma_L \\ \sigma_L \\ \sigma_L \end{pmatrix}} \lim_{\substack{\theta_L \\ \sigma_L \\ \sigma_L \\ \sigma_L \end{pmatrix}} \lim_{\substack{\theta_L \\ \sigma_L \\ \sigma_L$ small enough. Q.E.D.

Proof of Proposition 5. Note that, similar to the analysis in Section 5, the residual organ supply for category-2 candidates, μ_2 , is not enough to meet the total demand: $\mu_2 =$ $\sum_{i \in \{L,H\}} \Theta_i p_2 - \frac{\tau_2 + \delta_2}{\delta_2} \sum_{i \in \{L,H\}} [\Theta_i - \Phi_i F(C_i^{\sharp})].$ Thus, under the donor-priority rule, the arrival rate of category-2 priority candidates is $\lambda_{2p} = \sum_{i=H,L} \Theta_i p_2 F(C_i^{\sharp}) e^{-(\Theta_i + \sigma_i)S}$, and the arrival rate of category-2 nonpriority candidates is $\lambda_{2n} =$ $\sum_{i=H,L} \Theta_i p_2 [1 - F(C_i^{\sharp})e^{-(\theta_i + \sigma_i)^{\frac{1}{5}}}]$. Each priority candidate comes before nonpriority candidates of the same category in accessing the organs, meaning the total supply rate of organs available to donors μ_{2p} is the same as μ_2 , whereas the residual total supply rate of organs available to category-2 nonpriority candidates is $\mu_{2n} = \mu_2 - \lambda_{2p} = \sum_{i \in \{L,H\}} \Theta_i p_2 [1 - F(C_i^{\sharp})e^{-(\theta_i + \sigma_i)S}] - \frac{\tau_2 + \delta_2}{\delta_2} \sum_{i \in \{L,H\}} [\Theta_i - \Phi_i F(C_i^{\sharp})]$. By using (1) and (2), we represent each nonpriority candidate's pretransplantation life expectancy and probability of receiving an organ transplant as $D_{2n} = \frac{\sum_{i \in [L,H]} [\Theta_i - \Phi_i F(C_i^*)]}{\sum_{i \in [L,H]} \Theta_i p_2 [1 - F(C_i^*) e^{-(\theta_i + \alpha_i)S}]} \cdot \frac{\tau_2 + \delta_2}{\delta_2} \cdot d_2,$ and $\pi_{2n} = 1 - \frac{\sum_{i \in [L,H]} [\Theta_i - \Phi_i F(C_i^*)]}{\sum_{i \in [L,H]} \Theta_i p_2 [1 - F(C_i^*) e^{-(\theta_i + \alpha_i)S}]} \cdot \frac{\tau_2 + \delta_2}{\delta_2},$ respectively. Thus, a type-*i* individual, by not registering to become an organ donor, derives a utility of

$$\begin{split} U_n^i &= \frac{1}{\theta_i + \sigma_i} + \frac{\theta_i}{\theta_i + \sigma_i} \\ & \cdot \left\{ \beta T_p^{\sharp} - \frac{\sum_{j \in \{L,H\}} [\Theta_j - \Phi_j F(C_j^{\sharp})]}{\sum_{j \in \{L,H\}} \Theta_j [1 - F(C_j^{\sharp}) e^{-(\theta_j + \sigma_j)S}]} (\beta T_p^{\sharp} - \alpha/\delta_2) \right\}. \end{split}$$

By registering to become an organ donor, a type-i individual with a cost c derives a utility of

$$\begin{split} U_d^i(c) &= \frac{1}{\theta_i + \sigma_i} + \frac{\theta_i}{\theta_i + \sigma_i} \left\{ \beta T_p^{\sharp} - [1 - e^{-(\theta_i + \sigma_i)S}] \right. \\ &\left. \cdot \frac{\sum_{j \in \{L,H\}} [\Theta_j - \Phi_j F(C_j^{\sharp})]}{\sum_{j \in \{L,H\}} \Theta_j [1 - F(C_j^{\sharp}) e^{-(\theta_j + \sigma_j)S}]} \cdot (\beta T_p^{\sharp} - \alpha/\delta_2) \right\} - c. \end{split}$$

In equilibrium, a type-*i* individual with a threshold of $c_i = C_i^{\sharp}$, i = H, L is indifferent between joining the donor registry and not. In other words, $U_d^i(C_i^{\sharp}) = U_n^i$, which gives

$$C_{i}^{\sharp} = \frac{\theta_{i}}{\theta_{i} + \sigma_{i}} e^{-(\theta_{i} + \sigma_{i})S} \frac{\sum_{j \in \{L,H\}} (\Theta_{j} - \Phi_{j}F(C_{j}^{\sharp}))}{\sum_{j \in \{L,H\}} \Theta_{j} [1 - F(C_{j}^{\sharp})e^{-(\theta_{j} + \sigma_{j})S}]} \cdot (\beta T_{p}^{\sharp} - \alpha/\delta_{2}).$$

Similar to the proof of Proposition 3, we can show this equilibrium is unique. *Q.E.D.*

Proof of Corollary 9. The result follows by using $T_p(C_H^{\sharp}, C_L^{\sharp}) = \frac{\sum_{i=H,L} \Phi_i F(C_i^{\sharp}) T_i}{\sum_{i=H,L} \Phi_i F(C_i^{\sharp})}$ and $C_H^{\sharp} = C_L^{\sharp}$. *Q.E.D.*

Proof of Proposition 6. We observe that a type-*i* individual with the threshold C_i^{\sharp} , i = H, L is indifferent between joining the organ-donor registry and not joining, which suggests $U_d^i(C_i^{\sharp}) = U_n^i$. Plugging the expression of C_i^{\sharp} (see Proposition 5) into U_n^i , gives $U_d^i(C_i^{\sharp}) = U_n^i = \frac{1}{\theta_i + \sigma_i} + \frac{\theta_i}{\theta_i + \sigma_i} \beta T_p^{\sharp} - \frac{C_i^{\sharp}}{e^{-(\theta_i + \sigma_i)S}}$. Hence, we can represent social welfare under the donor-priority rule with a freeze-period remedy as

$$W_{p} = \underbrace{\sum_{i=H,L} \Lambda_{i} \int_{-\infty}^{C_{i}^{\sharp}} U_{d}^{i}(c)f(c)dc}_{\text{Donors'welfare}} + \underbrace{\sum_{i=H,L} \Lambda_{i} \int_{C_{i}^{\sharp}}^{\infty} U_{n}^{i}f(c)dc}_{\text{Non-donors' welfare}}$$
$$= \sum_{i=H,L} \Lambda_{i} \bigg[\frac{1}{\theta_{i} + \sigma_{i}} + \frac{\theta_{i}}{\theta_{i} + \sigma_{i}} \beta T_{p}^{\sharp} - \frac{C_{i}^{\sharp}}{e^{-(\theta_{i} + \sigma_{i})S}}$$
$$+ C_{i}^{\sharp}F(C_{i}^{\sharp}) - \int_{-\infty}^{C_{i}^{\sharp}} cf(c)dc \bigg].$$

When $S = S^{QR}$, we have $C_L^{\sharp} = C_H^{\sharp} = C_{OR}^{\sharp}$ and $T_p^{\sharp} = T_a$. Hence,

$$\begin{split} W_{p,QR} &= \sum_{i=H,L} \Lambda_i \bigg[\frac{1}{\theta_i + \sigma_i} + \frac{\theta_i}{\theta_i + \sigma_i} \beta T_a - \frac{C_{QR}^{\sharp}}{e^{-(\theta_i + \sigma_i)S^{QR}}} \\ &+ F(C_{QR}^{\sharp}) C_{QR}^{\sharp} - \int_{-\infty}^{C_{QR}^{\sharp}} cf(c) dc \bigg], \end{split}$$

with $\frac{\partial W_{p,QR}^{h}}{\partial C_{QR}^{\sharp}} = \sum_{i=H,L} \Lambda_{i} \left[F(C_{QR}^{\sharp}) - e^{(\theta_{i} + \sigma_{i})S^{QR}} \right]$, which is negative because $e^{(\theta_{i} + \sigma_{i})S^{QR}} > 1 > F(C_{QR}^{\sharp})$.

Note that, from Equation (7), we have

$$\begin{split} W_{np} &= \sum_{i \in \{L,H\}} \left\{ \Lambda_i \frac{1}{\theta_i + \sigma_i} + \Theta_i \beta T_a - [\Theta_i - \Phi_i F(C_i)] \right. \\ &\left. \cdot \left(\beta T_a - \alpha / \delta_2 \right) - \Lambda_i \int_{-\infty}^0 cf(c) dc \right\} \\ &= \sum_{i \in \{L,H\}} \Lambda_i \left\{ \frac{1}{\theta_i + \sigma_i} + \frac{\theta_i}{\theta_i + \sigma_i} \beta T_a - \left[\frac{\theta_i - \sigma_i \phi n F(C_i)}{\theta_i + \sigma_i} \right] \right. \\ &\left. \cdot \left(\beta T_a - \alpha / \delta_2 \right) - \Lambda_i \int_{-\infty}^0 cf(c) dc \right\}. \end{split}$$

Hence, $W_{p,QR} > W_{np}$ if and only if

$$\sum_{i=H,L} \Lambda_i \left[-\frac{C_{QR}^{\sharp}}{e^{-(\theta_i + \sigma_i)S^{QR}}} + F(C_{QR}^{\sharp})C_{QR}^{\sharp} - \int_0^{C_{QR}^{\sharp}} cf(c)dc \right]$$

> $-\sum_{i=H,L} \Lambda_i \frac{\theta_i - \sigma_i \phi nF(0)}{\theta_i + \sigma_i} (\beta T_a - \alpha/\delta_2),$

where $LHS > \sum_{i=H,L} \Lambda_i \left[F(0) - \frac{1}{e^{-(\theta_i + \sigma_i)S^{QR}}} \right] C_{QR}^{\sharp}$ because

$$\begin{split} F(C_{QR}^{\sharp})C_{QR}^{\sharp} &- \int_{0}^{C_{QR}^{\sharp}} cf(c)dc > F(C_{QR}^{\sharp})C_{QR}^{\sharp} \\ &- \int_{0}^{C_{QR}^{\sharp}} C_{QR}^{\sharp}f(c)dc = C_{QR}^{\sharp}F(0). \end{split}$$

By incorporating Proposition 5, which specifies the equilibrium C_i^{\sharp} and, thus, C_{OR}^{\sharp} , it suffices to show

$$\begin{split} & \frac{\sum_{i=H,L} \Lambda_i \frac{\theta_i}{\theta_i + \sigma_i} \left[F(0) e^{-(\theta_i + \sigma_i) S^{QR}} - 1 \right] \cdot \sum_{j \in \{L,H\}} \Lambda_j \frac{\theta_j - \sigma_j \phi n F(C_{QR}^{\flat})}{\theta_j + \sigma_j}}{\sum_{j \in \{L,H\}} \Lambda_j \frac{\theta_j}{\theta_j + \sigma_j} \left(1 - F(C_{QR}^{\flat}) e^{-(\theta_j + \sigma_j) S^{QR}} \right)} \\ & > - \sum_{i=H,L} \Lambda_i \frac{\theta_i - \sigma_i \phi n F(0)}{\theta_i + \sigma_i} \,, \end{split}$$

or equivalently,

$$\frac{\sum_{i \in \{L,H\}} \Lambda_i \frac{\sigma_i \phi_n - \theta_i e^{-(\theta_i + \sigma_i) S^{QR}}}{\theta_i + \sigma_i} F(C_{QR}^{\sharp})}{\sum_{i \in \{L,H\}} \Lambda_i \frac{\theta_i}{\theta_i + \sigma_i} \left[1 - F(C_{QR}^{\sharp}) e^{-(\theta_i + \sigma_i) S^{QR}}\right]} > \frac{\sum_{i \in \{L,H\}} \Lambda_i \frac{\sigma_i \phi_n - \theta_i e^{-(\theta_i + \sigma_i) S^{QR}}}{\theta_i + \sigma_i} F(0)}{\sum_{i = H,L} \Lambda_i \frac{\theta_i}{\theta_i + \sigma_i} \left[1 - F(0) e^{-(\theta_i + \sigma_i) S^{QR}}\right]}$$

which is true because $C_{QR}^{\sharp} > 0$. The reason is that the lefthand side can be considered a function of C_{QR}^{\sharp} , denoted by $\ell(C_{QR}^{\sharp})$. It is easy to show $\ell(C_{QR}^{\sharp})$ is increasing in C_{QR}^{\sharp} given $\sum_{i \in \{L,H\}} \Lambda_i \frac{\sigma_i \phi n - \theta_i e^{-(\theta_i + \sigma_i) S^{QR}}}{\theta_i + \sigma_i} > \sum_{i \in \{L,H\}} \Lambda_i \frac{\sigma_i \phi n - \theta_i}{\theta_i + \sigma_i} = \Phi - \Theta > 0$. Thus, we can obtain $LHS = \ell(C_{QR}^{\sharp}) > \ell(0) = RHS$. *Q.E.D.*

Proof of Proposition 7. First, as shown in Proposition 6, social welfare always increases under the quality-restoring freeze period $S^{QR} = \frac{\ln\left(\frac{\theta_H}{\theta_H + \sigma_H} / \frac{\theta_L}{\theta_H + \sigma_H} - \frac{\theta_L}{\theta_H + \sigma_H} - \frac{\theta_L}{\theta_H}\right)}{\theta_H + \sigma_H - \theta_L - \sigma_L}$. By definition, the optimal freeze period results in higher social welfare than the

quality-restoring freeze period. Next, social welfare under the donor-priority rule with $S = \infty$ is the same as that in the absence of the donor-priority rule. Therefore, a finite optimal freeze period exists, which, when combined with the donorpriority rule, always leads to increased social welfare. *Q.E.D.*

Endnotes

¹Because we focus on the broad implication of adopting the rule, we do not restrict ourselves to a specific type of organ (e.g., kidney, liver, heart, or tissue).

²Each deceased kidney donor can supply up to two kidneys.

³ For simplicity of analysis, we assume p_k to be independent of each individual's risk type. Our key findings extend to the case in which p_k is risk-type dependent under the condition that the initial distributions are not too different across risk types.

⁴ Countries implementing the donor-priority rule often require individuals to stay in the organ-donor registry for a freeze period before receiving donor priority. We consider the effect of imposing a freeze period for donor priority in Section 6. For now, we assume the following: (i) no transplant candidates will sign up as an organ donor; (ii) no individual, having become a donor, will renege from the donor registry; and (iii) eligible deaths of registered organ donors will lead to the harvest of organs. These assumptions ensure individuals signing up for the organ-donor registry will actually become suppliers of organs in the case of premature brain death.

⁵Our main findings hold qualitatively when such a distribution is specific to each individual's risk type.

⁶ Throughout the paper, we assume $\hat{C} > 0$ to be consistent with the observation from the organ-transplantation practice that the organdonation rates in the status quo are below the desired level; in the case of $\hat{C} < 0$, using policy initiatives to encourage organ donation is unnecessary.

⁷ The determinants of an individual's risk type are organ-specific. For example, in the case of the liver, research has shown that individuals with a history of long-term alcohol abuse, drug use, or receiving body piercings or tattoos using nonsterile instruments, among other risk factors, have a significantly higher likelihood of needing liver transplants (Centers for Disease Control and Prevention 2016, American Liver Foundation 2017).

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